Unit 2, Lesson 4

Combining Polynomials

Let's do arithmetic with polynomials.

4.1 Notice and Wonder: What Can Happen to Integers

What do you notice? What do you wonder?

- $7 \cdot 9 = 63$
- $7 + 9 = 16$
- $7 - 9 = \text{-}2$
- $\frac{7}{9} = 0.777\ldots$

4.2 Experimenting with Integers

Which of these statements are true? Give reasons in support of your answer.

1. If you add two even numbers, you'll always get an even number.

2. If you subtract an even number from another even number, you'll always get an even number.

3. If you add two odd numbers, you'll always get an odd number.

4. If you subtract an odd number from another odd number, you'll always get an odd number.

5. If you multiply two even numbers, you'll always get an even number.

6. If you multiply two odd numbers, you'll always get an odd number.

7. If you multiply two integers, you'll always get an integer.

8. If you add two integers, you'll always get an integer.

9. If you subtract one integer from another, you'll always get an integer.

Are you ready for more?

Which of these statements are true? Give reasons in support of your answer.

1. If you add two rational numbers, you'll always get a rational number.

2. If you multiply two rational numbers, you'll always get a rational number.

3. If you divide two rational numbers, you'll always get a rational number.

4.3 Experimenting with Polynomials

Here are some questions about polynomials. You and a partner will work on one of these questions.

1. If you add or subtract two polynomials, will you always get a polynomial? Explain your reasoning.

2. If you multiply two polynomials, will you always get a polynomial? Explain your reasoning.
   a. Try combining some polynomials to answer your question. Use the ones given by your teacher or make up your own. Keep a record of what polynomials you tried, and the results.
   b. When you think you have an answer to your question, explain your reasoning using equations, graphs, visuals, calculations, words, or in any way that will help others understand your reasons.

Lesson 4 Summary

If we add two integers, subtract one from the other, or multiply them, the result is another integer. The same is true for polynomials: Combining polynomials by addition, subtraction, or multiplication will always give us another polynomial.

For example, we can multiply $-x^2 + 4.5$ and $x^3 + 2x + \sqrt{7}$. We can use the distributive property and keep track of the results using a diagram like this:

	x^3	$2x$	$\sqrt{7}$
$-x^2$	$-x^5$	$-2x^3$	$-\sqrt{7}x^2$
4.5	$4.5x^3$	$9x$	$4.5\sqrt{7}$

Then we can find the product by adding all the results we filled in. This diagram tells us that the product is $-x^5 + 2.5x^3 - \sqrt{7}x^2 + 9x + 4.5\sqrt{7}$, which is also a polynomial even though there are square roots as coefficients! No matter what polynomials we start with, multiplying them will give us a polynomial. Adding or subtracting polynomials also gives us a polynomial, because we are just combining like terms.

When thinking about polynomials, it is important to remember exactly what counts as a polynomial. Any sum of terms that all have the same variable, where the variable is only raised to non-negative integer powers, is a polynomial. So some things that might not look like polynomials at first, such as -34.1 or $7.9998x$, are polynomials.