Multiplying Rational Numbers

Let's use signed numbers to represent movement.

14.1 Distance, Rate, Time

1. A car is traveling at a constant speed of 60 miles per hour. How far does the car travel in:
   a. 2 hours?

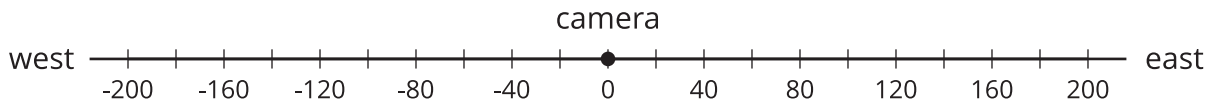
   b. 5 hours?

   c. x hours?

2. Create a representation that shows the relationship between the elapsed time and the distance traveled for this car.

14.2 Velocity

A traffic safety engineer was studying traffic patterns. She set up a camera to record the speed and direction of cars and trucks that passed by. She decided to represent positions to the east of the camera with positive numbers and positions to the west of the camera with negative numbers.



1. A car is traveling east at 12 meters per second. Where will it be 10 seconds after it passes the camera?

2. A car is traveling west at -14 meters per second. Where will it be 10 seconds after it passes the camera?

3. Complete the table to show the position of each vehicle after traveling at a constant velocity for the given amount of time.

	velocity (meters per second)	time after passing the camera (seconds)	position (meters)	equation
car A	+12	+10	+120	$12 \cdot 10 = 120$
car B	-14	+10		
car C	+9	+5		
car D	-11	+8		
car E	-15	+20		
car F	+8	0		

4. Complete the sentences. Be prepared to explain your reasoning.
    - A positive number times a positive number equals a _____.
    - A negative number times a positive number equals a _____.

Are you ready for more?

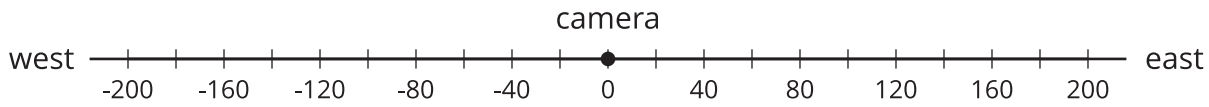
In many contexts we can interpret negative rates as "rates in the opposite direction." For example, a car that is traveling -35 miles per hour is traveling in the opposite direction of a car that is traveling 40 miles per hour.

1. What could it mean if we say that water is flowing at a rate of -5 gallons per minute?

2. Make up another situation with a negative rate, and explain what it could mean.

14.3 Backwards in Time

A traffic safety engineer was studying traffic patterns. She set up a camera to record the speed and direction of cars and trucks that passed by. She decided to represent positions to the east of the camera with positive numbers and positions to the west of the camera with negative numbers.



1. A car was traveling east at 12 meters per second. Where was the car 10 seconds *before* it passed the camera?

2. A car was traveling west at -14 meters per second. Where was the car 10 seconds *before* it passed the camera?

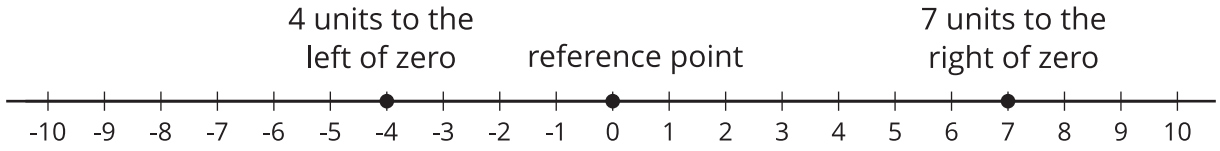
3. Complete the table to show the position of each vehicle after traveling at a constant velocity for the given amount of time.

	velocity (meters per second)	time after passing the camera (seconds)	position (meters)	equation
car A	+12	-10	-120	$12 \cdot -10 = -120$
car B	-14	-10		
car C	+9	-6		
car D	-11	-9		
car E	-15	-4		
car F	+8	-13		

4. Complete the sentences. Be prepared to explain your reasoning.
   - A positive number times a negative number equals a _____.
   - A negative number times a negative number equals a _____.

Lesson 14 Summary

We can use signed numbers to represent the position of an object along a line. We pick a point to be the reference point and call it zero. Positions to the right of zero are positive. Positions to the left of zero are negative.



When we combine speed with direction indicated by the sign of the number, it is called *velocity*. For example, if you are moving 5 meters per second to the right, then your velocity is +5 meters per second. If you are moving 5 meters per second to the left, then your velocity is -5 meters per second.

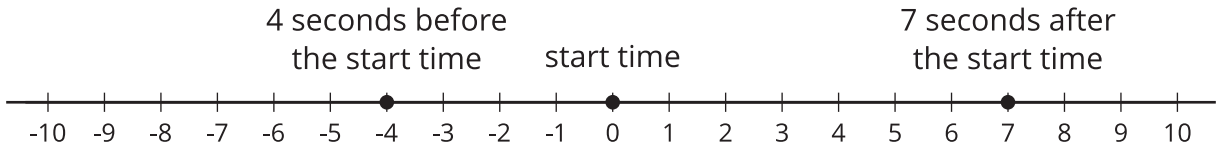
If you start at zero and move 5 meters per second for 10 seconds, you will be 50 meters to the *right* of zero. In other words,

$$5 \cdot 10 = 50$$

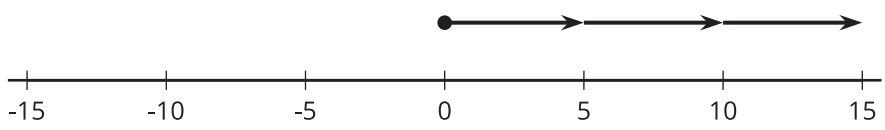
If you start at zero and move -5 meters per second for 10 seconds, you will be 50 meters to the *left* of zero. In other words,

$$-5 \cdot 10 = -50$$

We can also use signed numbers to represent time relative to a chosen point in time. We can think of this as starting a stopwatch. The positive times are after the watch starts, and negative times are times before the watch starts.

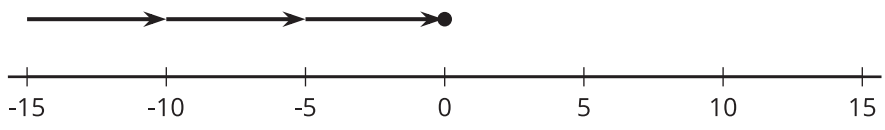


If a car is at position 0 and is moving in a positive direction, then for times after that (positive times), it will have a positive position.



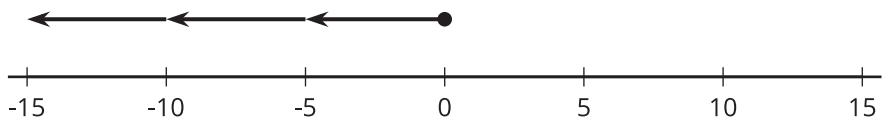
$$5 \cdot 3 = 15$$

For times *before* that (negative times), it must have had a negative position.



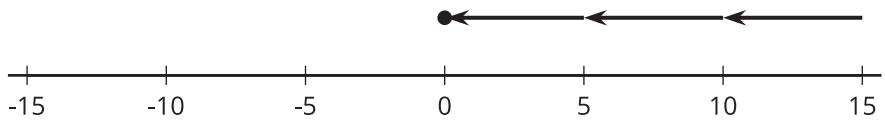
$$5 \cdot -3 = -15$$

If a car is at position 0 and is moving in a negative direction, then for times after that (positive times), it will have a negative position.



$-5 \cdot 3 = -15$

For times *before* that (negative times), it must have had a positive position.



$-5 \cdot -3 = 15$