



Causal Relationships

Goals

- Describe (orally and in writing) the strength and sign of the relationship between two variables.
- Investigate the relationship between two variables to analyze whether or not the relationship is causal.

Learning Targets

- I can look for connections between two variables to analyze whether or not there is a causal relationship.

Lesson Narrative

The mathematical purpose of this lesson is to understand that the relationship between variables can be, but is not always, a *causal relationship*. A **causal relationship** is one in which a change in one of the variables directly causes a change in the other variable.

Students have opportunities to refine their language to be more precise (MP6) as they describe the relationships. They are also using aspects of mathematical modeling (MP4) when they analyze the relationships between the variables.

Standards

Building On	HSS-ID.B.6
Addressing	HSS-ID.B.6, HSS-ID.C.9
Building Toward	HSS-ID.C.9

Instructional Routines

- Aspects of Mathematical Modeling
- MLR1: Stronger and Clearer Each Time

Student Facing Learning Goals

- Let's get a closer look at related variables.

9.1 Used Car Relationships

Warm-up

5 min

Activity Narrative

The mathematical purpose of this activity is for students to describe the relationship between variables using mathematical terminology, such as “strong relationship” or “weak relationship” and “positive relationship” or “negative relationship.” These terms were defined in the previous lesson. Students must reason abstractly and quantitatively (MP2) to determine the type of relationship.

Standards

Addressing	HSS-ID.B.6
Building Toward	HSS-ID.C.9



Launch

Arrange students in groups of 2. Tell students there are many possible answers for the questions. After quiet work time, ask students to compare their responses to their partner's and decide if they are both correct, even if they are different. Follow with a whole-class discussion.

To help students understand some of the context, explain that for many modern cars, it is recommended that the oil be changed every 5,000 miles driven or every 5 months.

Student Task Statement

Describe the strength and sign of the relationship you expect for each pair of variables. Be prepared to explain your reasoning.

1. Used car price and original sale price of the car.
2. Used car price and number of cup holders in the car.
3. Used car price and number of oil changes the car has had.
4. Used car price and number of miles the car has been driven.

Student Response

Sample response:

1. Strong, positive relationship. A car that cost a lot at first is probably still worth more later than a car that was less expensive when it was first bought.
2. Weak to no relationship. Although it is useful to have some cup holders, very little of a car's value is based on the number present in the car.
3. Strong, negative relationship. Since cars usually have their oil changed every few thousand miles or months, a lot of oil changes indicates the car has either been driven very far or owned for a long time, so it is probably worth less than a car with fewer oil changes.
4. Strong, negative relationship. Since the number of miles a car has been driven usually indicates more wear on the engine and other car parts, a car with a lot of miles is not worth as much as a car that has been driven less.

Activity Synthesis

The purpose of this discussion is for students to discuss the strength of relationships in preparation for having students distinguish between causal relationships and statistical relationships.

Encourage the use of the terms “strong relationship” or “weak relationship” and “positive relationship” or “negative relationship” in the discussion.

The discussion should focus on the reasoning for the last two problems and how they are similar and different. Here are some questions for discussion.

- “How is the relationship between the car price and number of oil changes similar to the relationship between the car price and number of miles driven?” (They both have a strong, negative relationship.)
- “How is the relationship between the car price and number of oil changes different from the relationship between the car price and number of miles driven?” (The number of miles the car has been driven seems more directly related to the price than the number of oil changes. It probably has a stronger negative relationship than the number of oil changes.)



Activity Narrative

The mathematical purpose of this activity is for students to describe how two variables are related, and to determine whether or not there is a causal relationship. Students should begin to recognize that while some variables may be related, one does not cause the other to change. At this point, the mathematics of scatter plot analysis cannot lead to a conclusion of whether there is a causal relationship. The relationship must be thought through carefully before a conclusion is made about whether the related variables in a situation have a causal relationship. As students analyze the relationships, they are modeling with mathematics (MP4).

Standards

Building On HSS-ID.B.6
Addressing HSS-ID.C.9

Instructional Routines

- Aspects of Mathematical Modeling

Launch

Arrange students in groups of 2. Tell students there are many possible answers for the questions.

Ask students what they know about rings inside a tree. If necessary, explain that every year a tree grows, the new growth appears as a ring in the trunk of the tree. The age of the tree can be found by counting the number of rings.

After quiet work time, ask students to compare their responses to their partner's and decide if they are both correct, even if they are different. Follow with a whole-class discussion.

Access for Students with Disabilities

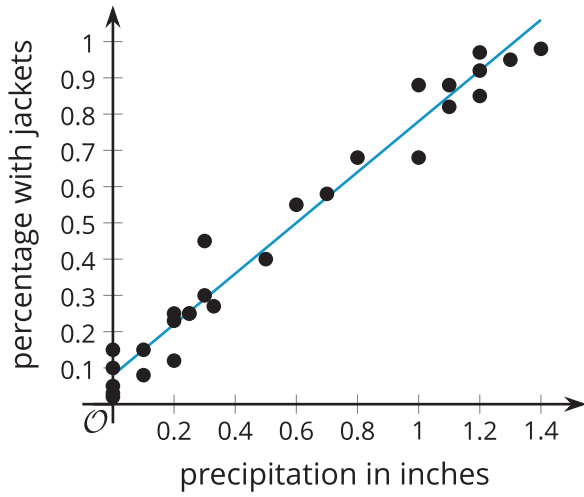
Representation: Develop Language and Symbols. Make connections between representations visible. After the first question, invite students to suggest language or annotations to include that will support their understanding of causal relationships. For example, write "Increased rain causes people to wear jackets," and use two different colors to highlight the axes and the variables in the phrase.

Supports accessibility for: Conceptual Processing, Language

Student Task Statement

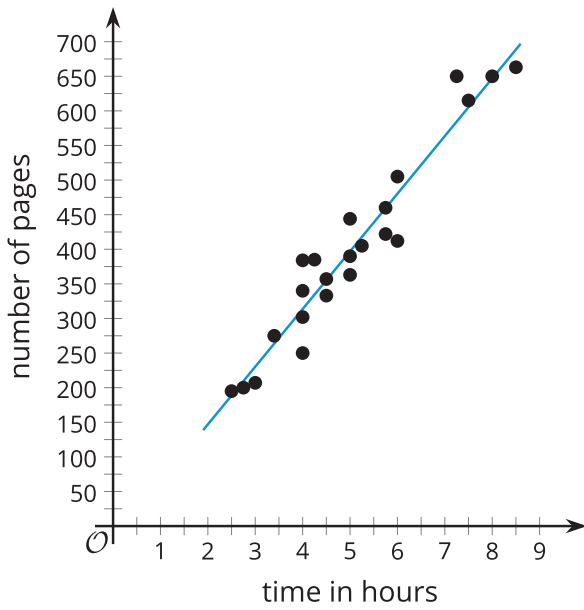
Each of the scatter plots show a strong relationship. Write a sentence or two describing how you think the variables are related.

1.



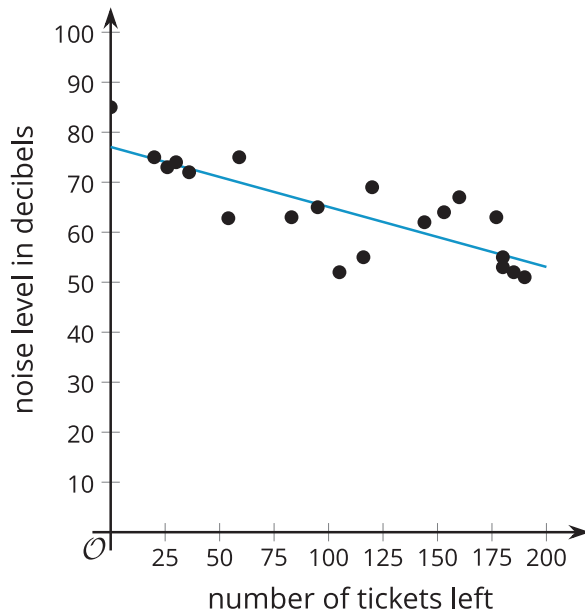
During the month of April, Elena keeps track of the number of inches of rain recorded for each day and the percentage of people who come to school with rain jackets on that day.

2.



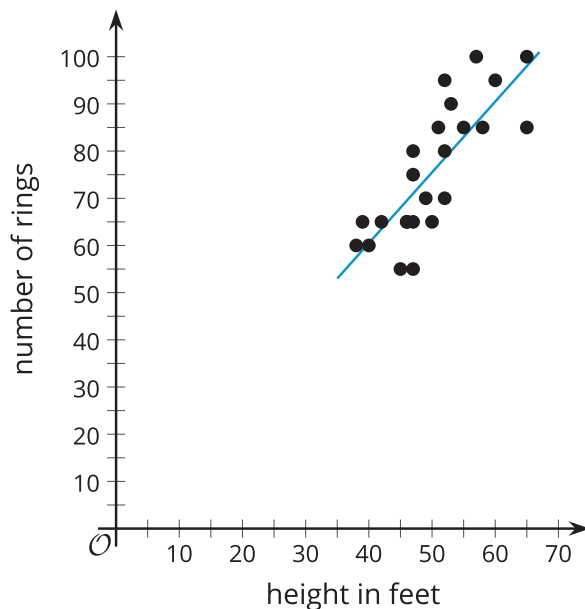
A school book club has a list of 100 books for its members to read. They keep track of the number of pages in each book that the members read from the list and the amount of time it took to read each book.

3.



A venue hosts holiday parties. On the day of each party, they count the tickets remaining and the noise level at the party.

4.



Pine trees grow in a forest. An arborist measures the height, in feet, of trees in a pine forest and counts the number of rings found in core samples from each tree.

Student Response

Sample responses:

1. When there is more rain, it is more likely people will wear jackets.
2. Longer books will take more time to read.
3. When more tickets are left, the party has fewer people, and therefore, they do not need to speak so loudly to hear one another.
4. Pine trees that are taller are usually older, so they would have more rings.

Activity Synthesis

The goal is for students to develop an understanding of what it means for a relationship between two variables to be *causal*, a term that will be introduced more formally in the next activity.

Select several students to share their reasoning for the relationship between the variables. For each pair of variables, ask students what might have *caused* the variables to be linked. In some cases (the first two here), one of the variables causes a change in the other. In other cases (the last two here), an additional variable or situation is the cause of the change.

Tell students that, since most people are used to seeing independent variables on the x -axis and dependent variables on the y -axis, the convention is to put the variable that caused the change on the x -axis and the other variable on the y -axis. The book club scatter plot should probably have the axes switched to meet the convention.

Explain that to truly determine causality, a careful experiment should be arranged to ensure other factors are not the source of change. Without those additional experiments, a more precise description would be to say that the amount of rain is an influential variable on jacket wearing.

Here are some questions for discussion.

- “Why does an increase in precipitation cause an increase in the percentage of people wearing rain jackets?” (When it rains, people usually wear coats to stay dry.)
- “An increase in the time it takes to read a book does not cause the number of pages in the book to increase. Does an increase in the number of pages in a book cause the time it takes to read a book to increase? Explain your reasoning.” (Yes, if you switch the axes on the graph, you can see that relationship. It is causal because it takes longer to read more pages.)
- “The relationship shown in the graph of the height of a pine tree and the number of tree rings appears to be strong. Does an increase in height cause an increase in the number of tree rings?” (It is not the height that causes the increase in rings, but probably the age of the tree that causes the increase. Age is positively related to height, so that likely explains the relationship seen between the two variables.)

9.3

Find Your Cause

🕒 10 min

Activity Narrative

The mathematical purpose of this activity is for students to practice thinking about the idea of a causal relationship. They think of situations in which the relationship is causal and those in which it is not.

This activity is a good chance to connect mathematical learning with learning happening in other subjects. Collaborate with other subject teachers to collect categories or quantities that could be related either causally or not.

Standards

Addressing HSS-ID.C.9

Instructional Routines

- MLR1: Stronger and Clearer Each Time

Launch

Tell students that “people often use the phrase ‘correlation, not causation’ or ‘association, not causation’ to refer to these situations in which there is a relationship, but it is not a causal relationship. A **causal relationship** means that a



change in one of the variables actually *causes* a change in the other variable."

Arrange students in groups of 2–4. If there are variables collected from other subjects, display them for students to consider for the activity.

Give students several minutes to record answers to the questions individually, and then ask students to share their answers with each other and determine if the answer is reasonable.



Access for English Language Learners

MLR1 Stronger and Clearer Each Time. Before the whole-class discussion, give students time to meet with 2–3 partners to share and get feedback on their first draft response to why their chosen variables fit the given categories. Invite listeners to ask questions and give feedback that will help their partner clarify and strengthen their ideas and writing. Give students 3–5 minutes to revise their first draft based on the feedback they receive.

Advances: Writing, Speaking, Listening



Access for Students with Disabilities

Engagement: Develop Effort and Persistence. Encourage and support opportunities for peer collaboration. Prior to directing student attention to the task, invite students to brainstorm various pairs of variables (both with correlation and without). Encourage students to free-associate and build off of each other's ideas. When students share their work with a partner, display sentence frames to support conversation, such as: "____ reminds me of ____ because . . ." and "____'s idea reminds me of . . ." After students have developed a list of several options, share the conditions list, and then encourage them to analyze and select which one best fits each condition.

Supports accessibility for: Language, Social-Emotional Functioning



Student Task Statement

Describe a pair of variables with each condition. Explain your reasoning.

1. Two variables have a **causal relationship**.
2. The variables are strongly related, but a third factor might be the cause for the changes in the variables.
3. The variables are only weakly related.

Student Response

Sample responses:

1. Length of a people's feet in inches and their men's shoe size. Increased foot size would cause someone to purchase larger shoe sizes.
2. Amount of power produced by solar panels and percentage of people wearing sunglasses while driving. These factors are probably strongly, positively related, but the cause is the amount of sunlight getting through to the ground for each variable rather than either one of the variables causing the other to increase directly.
3. The number of hits by a certain baseball player per game for a season and the price of a barrel of oil on the day of the games. There is no reasonable relationship between the variables, and they are very unlikely to be related even by external causes.

Building on Student Thinking

Students may still wonder how two variables can be correlated without having a causal relationship. It may help to



provide an example, such as sales of ice cream or sales of sunburn remedies. Ask students why these variables might be related and whether increasing one would cause the other variable to increase. Ask students to think of something that might cause two distinct outcomes to result.



Are You Ready for More?

1. Look through news articles or advertisements for claims of causation or correlation. Find 2 or 3 claims, and read or watch the articles or the advertisement. Answer these questions for each of the claims.
 - a. What is the claim?
 - b. What evidence is provided for the claim?
 - c. Does there appear to be evidence for causation or correlation? Explain your thinking
2. Choose the claim with the least or no evidence. Describe an experiment or other way that you could collect data to show correlation or causation.

Extension Student Response

Sample responses:

1.
 - a. Claim 1: Green tea lowers blood pressure. Claim 2: Chocolate improves health. Claim 3: Soda helps with heartburn.
 - b. Claim 1: Study groups were given either green tea or a drink that was colored like green tea for several weeks, and their blood pressure was taken before and after the study. Claim 2: They analyzed the number of antioxidants in chocolate compared with the number in fruit. Claim 3: No evidence was provided other than that it seemed to work for some people.
 - c. Claim 1: There appears to be evidence for correlation because there was a control group and the blood pressure was taken before and after drinking tea. However, there was no mention of how the groups were randomized or selected so I am not sure if there is evidence for causation. Claim 2: The only evidence provided is the quantity of antioxidants it contains, so a study would need to be done on antioxidants and health to show correlation or causation. Claim 3: There is no evidence for correlation or causation because only a testimonial is provided.
2. To show that heartburn relief is correlated with or caused by drinking soda, I would have to randomly assign subjects to different treatment groups. One group would get soda, and the other a placebo. I would then need to collect and compare data about their heartburn symptoms to determine if there is correlation or causation.

Activity Synthesis

The goal of this discussion is for students to gain a deeper understanding of what it means for variables to have a causal relationship.

Here are some questions for discussion.

- “Which pair of variables was the most difficult for you to describe? Explain your reasoning.” (I had a hard time describing a pair of variables that were strongly related but where a third factor might be the cause. It was difficult because I kept coming up with causal relationships.)
- “For question 1, how did you convince yourself or your group that one variable causes a change in the other?” (To convince them, I gave an example of using the variables in context. My two variables were snail weight and shell volume. When a snail has more weight, it increases in volume, which means it needs a bigger shell.)



Lesson Synthesis

Here are some questions for discussion.

- “How can you determine if there is a causal relationship between two variables? Explain your reasoning.” (To determine a causal relationship, you need to think about the context and determine if a change in one variable causes the other variable to change. One way to help determine whether there is a causal relationship is to design an experiment that controls one of the variables.)
- “Mai states that the relationship between the number of miles driven in a taxi and the price of the taxi ride is a causal relationship. Do you agree with Mai? What other information would help to further convince you one way or the other?” (I agree with her. It makes sense that the farther you go in a taxi, the more you will be charged. It might help convince me to take several taxi rides of the same length but with different starting points or destinations and see if the cost is the same.)
- “Jada states that the relationship between the size of a pasture and the number of cows kept at various farms is a causal relationship. Do you agree with Jada? Explain your reasoning.” (I do not agree. The increase in pasture size does not cause an increase in the number of cows [nor does an increase in number of cows make the pasture larger]. The increase in land might mean there is more food for the cows, but it is the farmer who decides how many cows there are on a farm.)

9.4 Just Cause

Cool-down

🕒 5 min

Standards

Addressing HSS-ID.C.9

Student Task Statement

For each pair of variables, decide whether you think there is:

- A very weak or no relationship.
- A strong relationship that is not a causal relationship.
- A causal relationship.

Explain your reasoning.

1. number of snow plows owned by a city and mitten sales in the city
2. number of text messages sent per day by a person and number of shirts owned by the person
3. price of a set of crayons and size of the box holding the crayons
4. amount of gas used on a trip and number of miles driven on the trip

Student Response

Sample responses:

1. A strong relationship that is not a causal relationship. The variables are related since cities with more snow plows will probably also have high sales of mittens compared to places with fewer snow plows. The climate, number of



people living in the city, and amount of snow all affect both of these variables. A city having extra snow plows would not cause people to buy more mittens, nor would the reverse happen.

2. A very weak or no relationship. These variables seem unrelated, and there is not another variable, like age or wealth, that seems to be related to both of these variables consistently.
3. A strong relationship that is not a causal relationship. The variables are related because a more expensive set of crayons will generally have more crayons for more color options, which requires a larger box, but it is the number of crayons that is the cause for the increase in both variables.
4. Causal relationship. Longer trips will cause greater gas consumption, and shorter trips will require less gas.

Responding to Student Thinking

Points to Emphasize

If most students struggle to identify causal relationships, consider using the optional lesson referred to here to provide additional practice discussing the issue. For example, discuss whether students think there is probably a direct link between penalties and wins or whether it might be a third variable that affects both variables.

Algebra 1, Unit 3, Lesson 10, Activity 2 Playing Dirty

Lesson 9 Summary

Humans are wired to look for connections and then use those connections to learn about the world around them. One way to notice connections is by looking for a pair of variables with a relationship. In order to learn about how the variables are related, we want to control one of the variables and see if there are changes in the other variable. For example, if we notice that people who tend to eat many calories also have a higher chance of having a heart attack, we might wonder if lowering our calorie intake would improve our health.

One common mistake people tend to make while using statistics is thinking that all relationships between variables are causal. Scatter plots can only show a relationship between the two variables. To determine if a change in one of the variables actually causes a change in the other variable, or if it has a **causal relationship**, the context must be better understood, and other options must be ruled out.

For example, we might expect to see a strong, positive relationship between the number of snowboard rentals and sales of hot chocolate during the months of September through January. This does not mean that an increase in snowboard rentals causes people to purchase more hot chocolate. Nor does it mean that increased sales of hot chocolate cause people to rent snowboards more. More likely there is a third variable, such as colder weather, that might be causing both variables to increase at the same time.

On the other hand, sometimes there is a causal relationship. A strong, positive relationship between hot chocolate sales and small marshmallow sales may be linked, because people buying hot chocolate may want to add small marshmallows to the drink, so an increase in the sales of hot chocolate are actually causing the marshmallow sale increase.

Finding relationships with the help of the correlation coefficient is a very good way to notice that there is a connection between variables. To determine whether the relationship is causal, the next step is usually to carefully design an experiment that isolates and precisely controls only one of the variables to determine how it affects the other variable.

Glossary

- causal relationship



Lesson 9 Practice Problems

1 Student Task Statement

Priya exercises and stops every so often to record the number of steps she has taken and her heart rate. When she gets home, she creates a scatter plot showing the relationship between the number of steps and her heart rate. The correlation coefficient for the data is 0.88.

- Describe the correlation as strong or weak and positive or negative. Explain your reasoning.
- Do you think either of the variables causes the other to change? Explain your reasoning.

Solution

- They are strongly, positively correlated. Sample reasoning: The correlation coefficient is close to 1. When Priya's steps increase, her heart rate increases.
- Sample response: It is likely that the type of exercise is the cause of both variables. For example, running on a treadmill would increase both the number of steps Priya takes and her heart rate. A long, slow walk would probably not raise her heart rate in the same way that a short, fast run would, even if the number of steps were the same.

2 Student Task Statement

Kiran creates a scatter plot showing the relationship between the number of students attending drama club and the number of students attending poetry club each week. The correlation coefficient for the data is -0.36.

- Describe the correlation as strong or weak and positive or negative. Explain your reasoning.
- Do you think either of the variables causes the other to change? Explain your reasoning.

Solution

- They are weakly, negatively correlated. Sample reasoning: The correlation coefficient is negative and near 0. It appears that as the number of students attending one of these clubs increases, the number of students attending the other club tends to decrease.
- Sample response: There is not evidence for causality. Any relationship between the variables is likely due to another cause, such as other events at the school conflicting with the club meeting time.

3 Student Task Statement

A news website shows a scatter plot with a negative relationship between the amount of sugar eaten and happiness levels. The headline reads, "Eating sugar causes happiness to decrease!"

- What is wrong with this claim?
- What is a better headline for this information?

Solution

- Scatter plots do not provide enough evidence for causality, only association.
- Sample response: A better headline might be, "Eating sugar is correlated with lower happiness levels, but we don't know why!"

4

from Unit 3, Lesson 1

Student Task Statement

A group of 125 college students are surveyed about their note-taking and study habits. Some results are represented in the table.

	prefer writing notes by hand	prefer typing notes	don't take notes
study for less than 1 hour	22		8
study for 1 hour or more	38	28	3

How many students prefer typing notes and study for less than 1 hour?

Solution

26

5

from Unit 3, Lesson 8

Student Task Statement

The number of miles driven, x , and the number of gallons remaining in the gas tank, y , have a strong, negative relationship.

Explain what it means to have a strong, negative relationship in this context.

Solution

Sample response: A strong relationship means that knowing the number of miles driven or the number of gallons remaining, along with the line of best fit, will produce a good estimate for the other variable. The negative relationship means that the number of gallons remaining in the tank decreases as the number of miles increases.

6

from Unit 3, Lesson 8

Student Task Statement

Technology required. Use a graphing calculator to a. What is an equation of the line of best fit?

answer the questions.

b. What is the value of the correlation coefficient?

x	y
10.2	31
10.4	27
10.5	29
10.5	30
10.5	31
10.6	26
10.8	25
10.8	26
10.9	27
11	24
11.2	22

Solution

- a. $y = -8.55x + 118.4$ (or equivalent)
b. $r = -0.86$