## Lesson 14: Completing the Square (Part 3)

* Let’s complete the square for some more complicated expressions.

### 14.1: Perfect Squares in Two Forms

Elena says, “$\left(x+3\right)^{2}$ can be expanded into $x^{2}+6x+9$. Likewise, $\left(2x+3\right)^{2}$ can be expanded into $4x^{2}+6x+9$.”

Find an error in Elena’s statement and correct the error. Show your reasoning.

### 14.2: Perfect in A Different Way

1. Write each expression in standard form:
	1. $\left(4x+1\right)^{2}$
	2. $\left(5x−2\right)^{2}$
	3. $\left(\frac{1}{2}x+7\right)^{2}$
	4. $\left(3x+n\right)^{2}$
	5. $\left(kx+m\right)^{2}$
2. Decide if each expression is a perfect square. If so, write an equivalent expression of the form $\left(kx+m\right)^{2}$. If not, suggest one change to turn it into a perfect square.
	1. $4x^{2}+12x+9$
	2. $4x^{2}+8x+25$

### 14.3: When All the Stars Align

1. Find the value of $c$ to make each expression in the left column a perfect square in standard form. Then, write an equivalent expression in the form of squared factors. In the last row, write your own pair of equivalent expressions.

| * standard form $\left(ax^{2}+bx+c\right)$
 | * squared factors $\left(kx+m\right)^{2}$
 |
| --- | --- |
| * $100x^{2}+80x+c$
 | *
 |
| * $36x^{2}−60x+c$
 | *
 |
| * $25x^{2}+40x+c$
 | *
 |
| * $0.25x^{2}−14x+c$
 | *
 |
| *
 | *
 |

1. Solve each equation by completing the square:
* $25x^{2}+40x=-12$
* $36x^{2}−60x+10=-6$

### 14.4: Putting Stars into Alignment

Here are three methods for solving $3x^{2}+8x+5=0$.

Try to make sense of each method.

Method 1:

$\begin{matrix}3x^{2}+8x+5&=0\\\left(3x+5\right)\left(x+1\right)&=0\end{matrix}$

$\begin{matrix}x=-\frac{5}{3} or x=-1\end{matrix}$

Method 2:

$\begin{matrix}3x^{2}+8x+5&=0\\9x^{2}+24x+15&=0\\\left(3x\right)^{2}+8\left(3x\right)+15&=0\\U^{2}+8U+15&=0\\\left(U+5\right)\left(U+3\right)&=0\end{matrix}$
$\begin{matrix}U=-5 &or U=-3\\3x=-5 &or 3x=-3\\x=-\frac{5}{3} &or x=-1\end{matrix}$

Method 3:

$\begin{matrix}3x^{2}+8x+5&=0\\9x^{2}+24x+15&=0\\9x^{2}+24x+16&=1\\\left(3x+4\right)^{2}&=1\end{matrix}$

$\begin{matrix}3x+4=1 &or 3x+4=-1\\x=-1 &or x=-\frac{5}{3}\end{matrix}$

Once you understand the methods, use each method at least one time to solve these equations.

1. $5x^{2}+17x+6=0$
2. $6x^{2}+19x=-10$
3. $8x^{2}−33x+4=0$
4. $8x^{2}−26x=-21$
5. $10x^{2}+37x=36$
6. $12x^{2}+20x−77=0$

#### Are you ready for more?

Find the solutions to $3x^{2}−6x+\frac{9}{4}=0$. Explain your reasoning.

### Lesson 14 Summary

In earlier lessons, we worked with perfect squares such as $\left(x+1\right)^{2}$ and $\left(x−5\right)\left(x−5\right)$. We learned that their equivalent expressions in standard form follow a predictable pattern:

* In general, $\left(x+m\right)^{2}$ can be written as $x^{2}+2mx+m^{2}$.
* If a quadratic expression of the form $ax^{2}+bx+c$ is a perfect square, and the value of $a$ is 1, then the value of $b$ is $2m$, and the value of $c$ is $m^{2}$ for some value of $m$.

In this lesson, the variable in the factors being squared had a coefficient other than 1, for example $\left(3x+1\right)^{2}$ and $\left(2x−5\right)\left(2x−5\right)$. Their equivalent expression in standard form also followed the same pattern we saw earlier.

| squared factors | standard form |
| --- | --- |
| $\left(3x+1\right)^{2}$ | $\left(3x\right)^{2}+2\left(3x\right)\left(1\right)+1^{2} or 9x^{2}+6x+1$ |
| $\left(2x−5\right)^{2}$ | $\left(2x\right)^{2}+2\left(2x\right)\left(-5\right)+\left(-5\right)^{2} or 4x^{2}−20x+25$ |

In general, $\left(kx+m\right)^{2}$ can be written as:

$\left(kx\right)^{2}+2\left(kx\right)\left(m\right)+m^{2}$

or

$k^{2}x^{2}+2kmx+m^{2}$

If a quadratic expression is of the form $ax^{2}+bx+c$, then:

* the value of $a$ is $k^{2}$
* the value of $b$ is $2km$
* the value of $c$ is $m^{2}$

We can use this pattern to help us complete the square and solve equations when the squared term $x^{2}$ has a coefficient other than 1—for example: $16x^{2}+40x=11$.

What constant term $c$ can we add to make the expression on the left of the equal sign a perfect square? And how do we write this expression as squared factors?

* 16 is $4^{2}$, so the squared factors could be $\left(4x+m\right)^{2}$.
* 40 is equal to $2\left(4m\right)$, so $2\left(4m\right)=40$ or $8m=40$. This means that $m=5$.
* If $c$ is $m^{2}$, then $c=5^{2}$ or $c=25$.
* So the expression $16x^{2}+40x+25$ is a perfect square and is equivalent to $\left(4x+5\right)^{2}$.

Let’s solve the equation $16x^{2}+40x=11$ by completing the square!

$\begin{matrix}16x^{2}+40x&=11\\16x^{2}+40x+25&=11+25\\\left(4x+5\right)^{2}&=36\\&\\4x+5=6 &or 4x+5=-6\\4x=1 &or 4x=-11\\x=\frac{1}{4} &or x=-\frac{11}{4}\end{matrix}$.



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