Writing Systems of Equations

Let’s write systems of equations from real-world situations.

15.1 How Many Solutions?

How many solutions does each system have? Be prepared to share your reasoning.

1. $\begin{cases} y = -\frac{4}{3}x + 4 \\ y = -\frac{4}{3}x - 1 \end{cases}$

2. $\begin{cases} y = 4x - 5 \\ y = -2x + 7 \end{cases}$

3. $\begin{cases} 2x + 3y = 8 \\ 4x + 6y = 17 \end{cases}$

4. $\begin{cases} y = 5x - 15 \\ y = 5(x - 3) \end{cases}$

15.2 Info Gap: Racing and Play Tickets

Your teacher will give you either a problem card, or a data card. Do not show or read your card to your partner.

If your teacher gives you the problem card:

1. Silently read your card and think about what information you need to answer the question.
2. Ask your partner for the specific information that you need. "Can you tell me ________?"
3. Explain to your partner how you are using the information to solve the problem. "I need to know ________ because . . . ."
   Continue to ask questions until you have enough information to solve the problem.
4. When you have enough information, share the problem card with your partner, and solve the problem independently.
5. Read the data card, and discuss your reasoning.

If your teacher gives you the data card:

1. Silently read your card. Wait for your partner to ask for information.
2. Before telling your partner any information, ask, "Why do you need to know ________?"
3. Listen to your partner's reasoning and ask clarifying questions. Give only information that is on your card. Do not figure out anything for your partner!
   These steps may be repeated.
4. When your partner says they have enough information to solve the problem, read the problem card, and solve the problem independently.
5. Share the data card, and discuss your reasoning.

15.3 Situations and Systems

For each situation:

- Create a system of equations.
- Then, without solving, interpret what the solution to the system would tell you about the situation.

1. Lin’s family is out for a bike ride when her dad stops to take a picture of the scenery. He tells the rest of the family to keep going and that he’ll catch up. Lin's dad spends 5 minutes taking the photo and then rides at 0.24 miles per minute until he meets up with the rest of the family further along the bike path. Lin and the rest were riding at 0.18 miles per minute.

2. Noah is planning a kayaking trip. Kayak Rental A charges a base fee of \$15 plus \$4.50 per hour. Kayak Rental B charges a base fee of \$12.50 plus \$5 per hour.

3. Diego is crafting items in a video game. The crafting recipe calls for 3 pieces of wood for every piece of metal. Diego’s character uses 52 item slots of wood and metal crafting materials while making the items.

4. Sunflower seeds for birds cost \$2.50 per pound and a generic birdseed costs \$1.25 per pound. An order of some of each type of bird seed weighs 15 pounds and costs \$25.00.

15.4 Solving Systems Practice

Here are a lot of systems of equations:

- $\begin{cases} y = \text{-}2x + 6 \\ y = x - 3 \end{cases}$
- $\begin{cases} y = 5x - 4 \\ y = 4x + 12 \end{cases}$
- $\begin{cases} y = \frac{2}{3}x - 4 \\ y = \text{-}\frac{4}{3}x + 9 \end{cases}$
- $\begin{cases} 4y + 7x = 6 \\ 4y + 7x = \text{-}5 \end{cases}$
- $\begin{cases} y = x - 6 \\ x = 6 + y \end{cases}$
- $\begin{cases} y = 0.24x \\ y = 0.18x + 0.9 \end{cases}$
- $\begin{cases} y = 4.5x + 15 \\ y = 5x + 12.5 \end{cases}$
- $\begin{cases} y = 3x \\ x + y = 52 \end{cases}$

1. Without solving, identify 3 systems that you think would be the least difficult for you to solve and 3 systems you think would be the most difficult. Be prepared to explain your reasoning.

2. Choose 4 systems to solve. At least one should be from your "least difficult" list and one should be from your "most difficult" list.

Lesson 15 Summary

We have learned how to use algebra to solve many kinds of systems of equations that would be difficult to solve by graphing. For example, look at

$$\begin{cases} y = 2x - 3 \\ x + 2y = 7 \end{cases}$$

The first equation says that $y = 2x - 3$, so wherever we see y, we can substitute the expression $2x - 3$ instead. So the second equation becomes $x + 2(2x - 3) = 7$.

We can solve for x:

$$\begin{aligned} x + 4x - 6 &= 7 && \text{distributive property} \\ 5x - 6 &= 7 && \text{combine like terms} \\ 5x &= 13 && \text{add 6 to each side} \\ x &= \frac{13}{5} && \text{multiply each side by } \frac{1}{5} \end{aligned}$$

We know that the y-value for the solution is the same for either equation, so we can use either equation to solve for it. Using the first equation, we get:

$$\begin{aligned} y &= 2\left(\frac{13}{5}\right) - 3 && \text{substitute } x = \frac{13}{5} \text{ into the equation} \\ y &= \frac{26}{5} - 3 && \text{multiply } 2\left(\frac{13}{5}\right) \text{ to make } \frac{26}{5} \\ y &= \frac{26}{5} - \frac{15}{5} && \text{rewrite 3 as } \frac{15}{5} \\ y &= \frac{11}{5} \end{aligned}$$

If we substitute $x = \frac{13}{5}$ into the other equation, $x + 2y = 7$, we get the same y value. So the solution to the system is $\left(\frac{13}{5}, \frac{11}{5}\right)$.