Logarithm Product Rule

Let's combine logarithms that are added together.

15.1 A Pattern in Logarithm Sums

For each sum, find the value of the sum by finding the value of each logarithm and then adding the values. Then complete the given logarithm so that it has the same value as the sum.

The first one is done for you. Discuss with your partner why it is true.

1. $\log_2(8) + \log_2(4) = \underline{5}$

   $\log_2(8) + \log_2(4) = \log_2(\underline{32})$

2. $\log_3(9) + \log_3(81) = \_\_\_\_\_\_\_$

   $\log_3(9) + \log_3(81) = \log_3(\_\_\_\_\_\_\_)$

3. $\log_3\left(\frac{1}{9}\right) + \log_3(81) = \_\_\_\_\_\_\_$

   $\log_3\left(\frac{1}{9}\right) + \log_3(81) = \log_3(\_\_\_\_\_\_\_)$

4. $\log_5(25) + \log_5(1) = \_\_\_\_\_\_\_$

   $\log_5(25) + \log_5(1) = \log_5(\_\_\_\_\_\_\_)$

5. $\log(100) + \log(1{,}000) = \_\_\_\_\_\_\_$

   $\log(100) + \log(1{,}000) = \log(\_\_\_\_\_\_\_)$

15.2 Making a Conjecture about Logarithm Sums

1. Use the pattern you noticed about sums of logarithms with the same base to write a conjecture.

$$\log_b(U) + \log_b(V) = \log_b\left(\underline{\hspace{3cm}}\right)$$

2. Assume the conjecture is true. Rewrite each expression as a single logarithm, then find its value.

   a. $\log_6(12) + \log_6(3)$

   b. $\log(250) + \log(40)$

   c. $\log_8\left(\frac{3}{64}\right) + \log_8\left(\frac{1}{3}\right)$

3. If $\log(3) = 0.4771$ and $\log(7) = 0.8451$, find the values of each logarithm. Explain or show your reasoning.

   a. $\log(30)$

   b. $\log(21)$

   c. $\log(27)$

15.3 Proving the Conjecture about Logarithm Sums

Let's work through some steps of a proof for your conjecture.

Start with two equations: $b^x = U$ $b^y = V$

1. Rewrite both of these equations as logarithms, and circle your answers to use later. $\log \quad (\qquad) \quad \log \quad (\qquad)$
2. Multiply the left sides of the original equations, and set the product equal to the product of the right sides of the original equations. $\underline{\qquad\qquad} = UV$
3. Combine the exponents on the left side of the equation so that it is written with a single base. $\underline{\qquad\qquad} = UV$
4. Rewrite the last equation as a logarithm. $\log \quad (\qquad)$
5. Use your circled equations to replace any x and y in that equation with equivalent logarithms.

Are you ready for more?

Before electronic calculators were readily available, people used this rule to multiply large numbers using logarithm tables such as the ones earlier in this unit. To see an example, research how to use a slide rule.

Lesson 15 Summary

The **product rule** for logarithms allows us to combine a sum of logarithms with the same base into a single logarithm. The product rule states that

$$\log_a(b) + \log_a(c) = \log_a(b \cdot c)$$

For example, $\log(6) + \log(5) = \log(30)$.

Thinking about logarithms in relation to exponents, this may make more sense. We learned in an earlier course that

$$a^x \cdot a^y = a^{x+y}$$

By rewriting parts of that equation into their logarithm form, we can combine the pieces to prove the product rule.

