Unit 7, Lesson 3

# Nonadjacent Angles

## Goals

- Comprehend that the term "vertical angles" (in spoken and written language) refers to a pair of angles created by two intersecting lines.
- Generalize (orally and in writing) that the opposite angles created by two intersecting lines have equal angle measures.
- Use reasoning about angle measures to identify complementary or supplementary angles that are not adjacent.

## Learning Targets

- I can determine if angles that are not adjacent are complementary or supplementary.
- I can explain what vertical angles are in my own words.

## Lesson Narrative

In this lesson, students see that angles do not need to be adjacent to be complementary or supplementary. Students are also introduced to and begin to use the term **vertical angles** for describing the opposite angles formed when two lines cross (MP6). They use repeated reasoning to see that the vertical angles have equal measures (MP8).

Students can relate this understanding to the fact that both angles in a pair of vertical angles are supplementary to the same angle in between, but students do not need to be able to give a formal geometric proof that vertical angles must have equal measures.

## Standards

Addressing **7.EE.A, 7.G.B.5**

## Instructional Routines

- MLR2: Collect and Display
- MLR8: Discussion Supports

## Required Materials

### Materials to Gather

- Geometry toolkits: Activity 2, Activity 3

## Student Facing Learning Goals

Let's look at angles that are not right next to one another.

# 3.1 Finding Related Statements

Warm-up

5 min

## Activity Narrative

The purpose of this *Warm-up* is for students to use structure to reason about equivalent equations. In this unit, students will write equations to represent how angles are related to each other, and this *Warm-up* helps prepare for that work.

All of the given statements *could* be true so students may be quick to say each of them must be true. Ask these students if there is a case in which that particular statement would not be true for possible values for $a$ and $b$.

## Standards

Addressing **7.EE.A**

## Launch

Arrange students in groups of 2.

Ask students, "If we know for sure that $a + b = 180$, what are some possible values of $a$ and $b$?" Give students 30 seconds of quiet think time, and then ask several students to share their responses. Some examples are $a = 90$ and $b = 90$, $a = 0$ and $b = 180$, and $a = 10$ and $b = 170$. Tell students that in this activity, we know for sure that $a + b = 180$, but we don't know the exact values of $a$ and $b$.

Give students 2 minutes of quiet work time followed by 1 minute to discuss their responses with a partner. Follow with a whole-class discussion.

## Student Task Statement

If $a + b = 180$ is true, which statements also must be true?

$a = 180 - b$ $\quad$ $a - 180 = b$ $\quad$ $360 = 2a + 2b$ $\quad$ $a = 90$ and $b = 90$

## Student Response

$a = 180 - b$ and $360 = 2a + 2b$

## Activity Synthesis

Invite students to share their reasoning for each statement. If students disagree, allow students to discuss until they come to an agreement. Consider asking some of the following questions while students discuss:

- "Do you have an example that might support this statement being true (or untrue)?"
- "What evidence do you have to support that statement being true (or untrue)?"
- "What other values of $a$ and $b$ might work?"
- "What was done to the equation to make the statement true (or untrue)?"

If students claim that $a - 180 = b$ or $a = 90$ and $b = 90$ must be true, explain that this *may* be true, but does not have to be true. Invite them to discuss this idea and think of examples where $a + b = 180$ but their statement is not true.

If students claim that $a - 180 = b$ cannot be true, ask them to consider what would happen if one of the values is 0.

# 3.2 Polygon Angles

10 min

## Activity Narrative

In this activity, students see that angles do not need to be adjacent to each other in order to be considered complementary or supplementary. Students are given two different polygons and are asked to find complementary and supplementary angles, using any tools in their geometry toolkit (MP5). The most likely approaches are:

- Measure each angle with a protractor and and look for any that add up to 180 or 90 degrees.
- Trace the legs of an angle with tracing paper and align its vertex and one leg with another angle to see if the two angles, when adjacent, form a straight angle or a right angle.

As students work, monitor for students who use either approach listed or some other strategy. Also, encourage students to use precise vocabulary and language that they learned in previous activities and lessons (MP6).

## Standards

Addressing **7.G.B.5**

## Instructional Routines

- MLR8: Discussion Supports

## Launch

Invite students to share their definitions of *complementary* and *supplementary*, and consider displaying the meanings for all to see through the remainder of the class. Highlight ideas that involve angle measurements or sums, and leave ideas implying that they must be touching or adjacent as an open question for now.

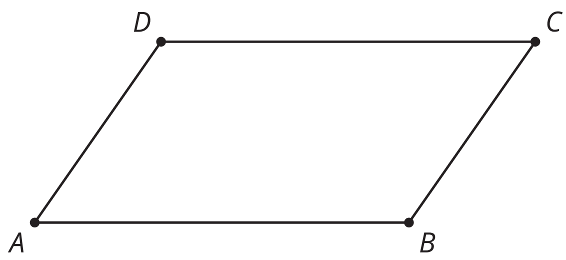
Arrange students in groups of 2. Provide access to geometry toolkits. Give students 3–4 minutes of quiet work time, followed by partner and whole-class discussions.

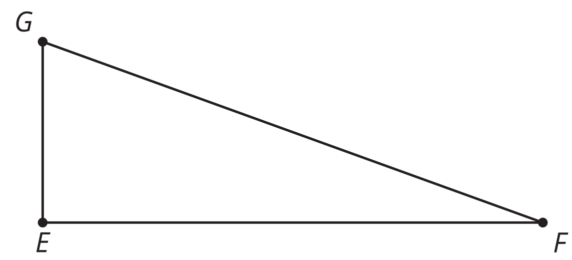
## Access for Students with Disabilities

*Action and Expression: Internalize Executive Functions.* To support organization, provide students with a printed enlarged version of the figures in the *Task Statement* to be able to measure the angles without extending the lines.
*Supports accessibility for: Language, Organization*

## Student Task Statement

Identify any pairs of angles in these figures that are complementary or supplementary.






### Student Response

- In the quadrilateral, there are four pairs of supplementary angles:
  - Angle $BAD$ is supplementary with angle $ABC$.
  - Angle $BAD$ is also supplementary with angle $ADC$.
  - Angle $BCD$ is supplementary with angle $ABC$.
  - Angle $BCD$ is also supplementary with angle $ADC$.
- In the triangle, there is one pair of complementary angles: angle $EFG$ is complementary with angle $EGF$.

### Building on Student Thinking

Some students may struggle to use a protractor to measure angles when the rays are not drawn long enough to reach the edge of the protractor. Prompt them to extend the sides of the angle, using a straightedge.

### Activity Synthesis

Invite previously selected students to share their strategies for finding a pair of complementary and supplementary angles. Ensure that correct use of a protractor to find the measure of an angle is clearly and carefully demonstrated. This will help all students prepare for the next activity, in which everyone will be using a protractor.

### Access for English Language Learners

*MLR8 Discussion Supports*. For each method for finding pairs of angles that is shared, invite students to turn to a partner and restate what they heard, using precise mathematical language. Consider asking the original speaker if their peer accurately restated their method.
*Advances: Listening, Speaking*

## 3.3 Vertical Angles

15 min

### Activity Narrative

The purpose of this activity is for students to learn about **vertical angles.** Each student draws two intersecting lines and measures the four resulting angles. Then, students examine multiple examples to come up with a conjecture for any relationships they noticed (MP8).

## Access for English Language Learners

This activity uses the *Collect and Display* math language routine to advance conversing and reading as students clarify, build on, or make connections to mathematical language.

## Standards

Addressing **7.G.B.5**

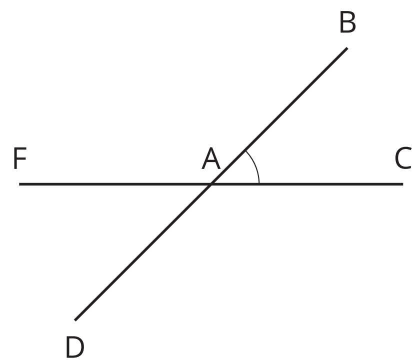
## Instructional Routines

- MLR2: Collect and Display

## Launch

Arrange students in groups of 2–4. Provide access to geometry toolkits.

Display this image for all to see. Tell students that this is a pair of *intersecting* lines, or lines that cross at a point.




Ask students to identify 2 angles that are supplementary to angle $BAC$ ($BAF$ and $CAD$). Invite a student to demonstrate measuring these angles with a protractor (135 degrees). If possible, leave the diagram with angle measures marked displayed throughout the activity.

Tell students they will draw their own diagram with a different set of intersecting lines. Remind them to draw arcs to label the degree measures of their angles.

Give students 2–3 minutes to draw and measure the figure. Remind them to draw arcs to label the degree measures of their angles. Each person in the group should draw a different diagram. Follow with small-group and whole-class discussions.

Use *Collect and Display* to direct attention to words collected and displayed from an earlier activity. Collect the language that students use to describe the relationships they notice between angles. Display words and phrases, such as "adjacent," "right," "straight," "complementary," and "supplementary," as well as descriptions such as "across from each other," "mirrors of each other," or "add up to a straight angle."

## Student Task Statement

Use a straightedge to draw two intersecting lines. Use a protractor to measure all four angles whose vertex is located at the intersection.

Compare your drawing and measurements to the drawings and measurements created by others in your group. What patterns do you notice about the relationships between angle measures at an intersection?

## Student Response

Sample response: "The angles across from each other will always be equal."

## Building on Student Thinking

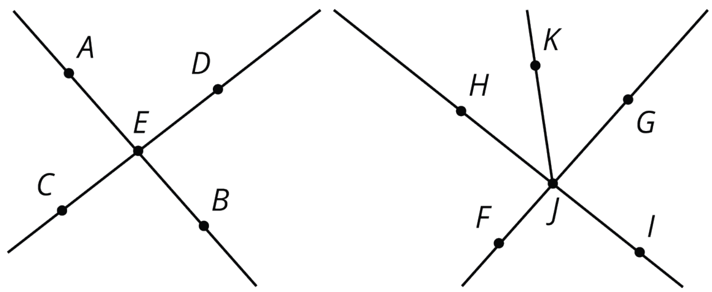
Some students may label the angle measures toward the end of the rays, where they read the number from the protractor. This is not precise enough, because two different angles share each ray. Remind students about drawing arcs to clarify which angle they measured.

## Activity Synthesis

Direct students' attention to the reference created using Collect and Display. Ask students to share their observations. Invite students to borrow language from the display as needed. As they respond, update the reference to include additional phrases. (For example, the display may have "The angles across from each other look the same" already on it, which can be updated with the more precise phrase "The opposite angles have the same angle measure.")
If there are students who use supplementary angles to explain why vertical angles have equal measures, ask them to share their observations last.

Define **vertical angles** as a pair of angles, formed by two intersecting lines, that are opposite each other.

Display the image, and ask students to identify four pairs of vertical angles. In particular, students may have trouble seeing that angles $FJI$ and $HJG$ are vertical angles.




Although students don't need to know a proof that vertical angles always have the same measure, it may be helpful to show one way to understand why they are. If time allows, share that in the image. . .

- Angles $AED$ and $AEC$ are supplementary, so the sum of their measures is 180 degrees.
- Angles $AEC$ and $CEB$ are also supplementary, so the sum of their measures is also 180 degrees.
- If we take angle $AEC$ away from the straight angles, we see that angles $AED$ and $CEB$ must have the same measure.

## 3.4 Row Game: Angles

Optional

10 min

## Activity Narrative

This activity gives students an opportunity to practice recognizing complementary, supplementary, and vertical angles and using what they know about those types of angles to find unknown angle measures. Some students may feel comfortable writing equations to show their reasoning, but it is not important that all students use this strategy at this point, as it will be the focus of future lessons. Encourage students to continue using the new vocabulary.

## Standards

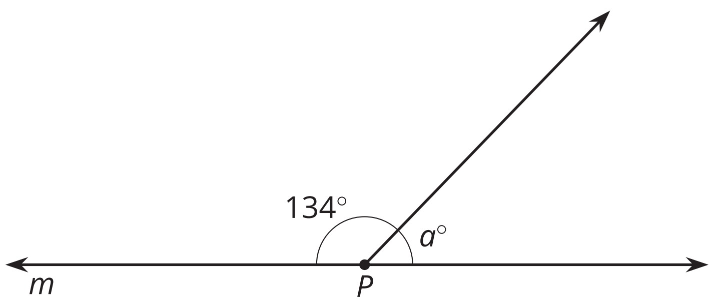
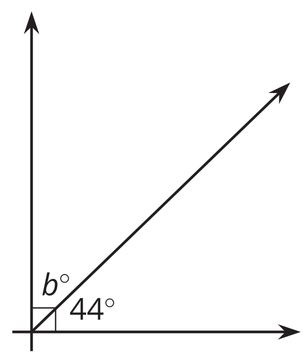
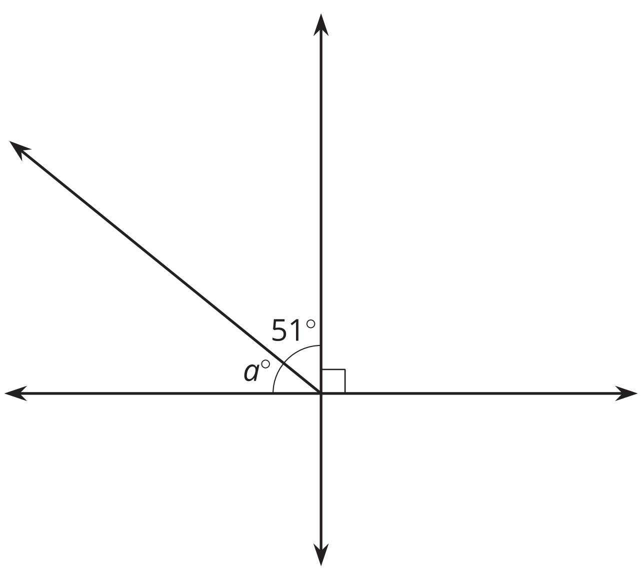
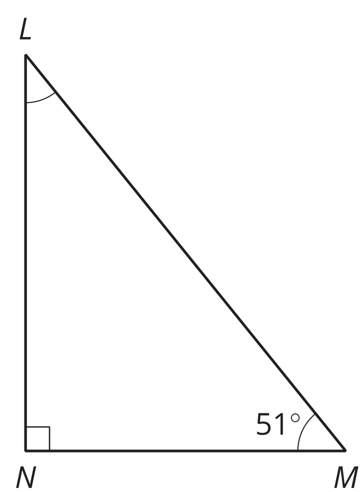
Addressing **7.G.B.5**

## Launch

Arrange students in groups of 2. Make sure students know how to play a row game. Give students 5–6 minutes of partner work time followed by a whole-class discussion.

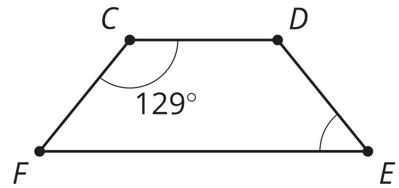
## Student Task Statement

Find the measurement of the angles in one column. Your partner will work on the other column. Check in with your partner after you finish each row. Your answers in each row should be the same. If your answers aren't the same, work together to find the error and correct it.

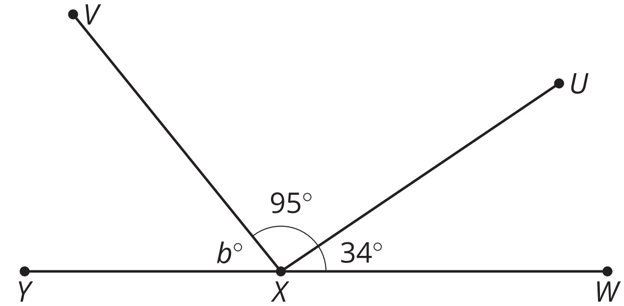
| column A | column B |
| --- | --- |
| $P$ is on Line $m$. Find the value of $a$.   | Find the value of $b$.   |
| Find the value of $a$.   | In right triangle $LMN$, angles $L$ and $M$ are complementary. Find the measure of angle $L$.   |

column A column B

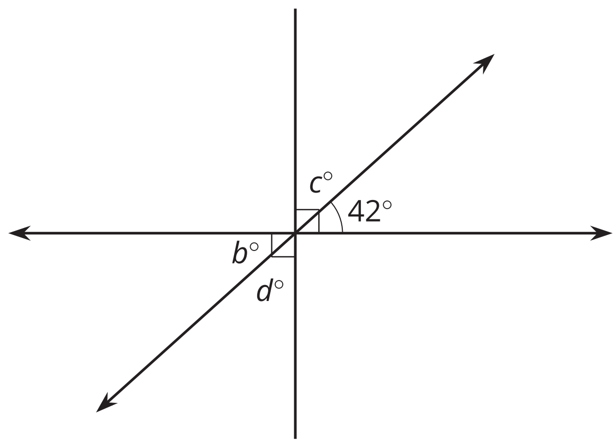
Angle $C$ and angle $E$ are supplementary. Find the measure of angle $E$.



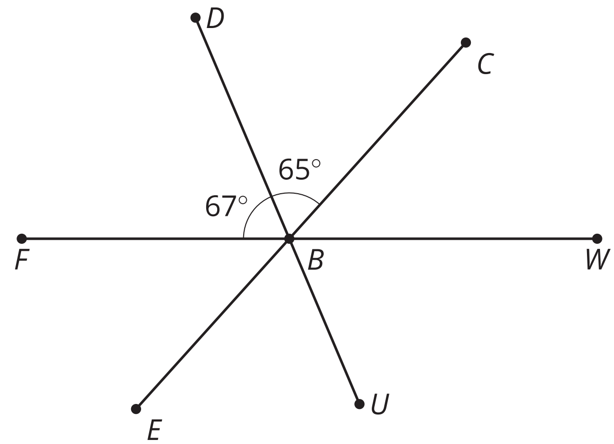

$X$ is on line $WY$. Find the value of $b$.




Find the value of $c$.




$B$ is on line $FW$. Find the measure of angle $CBW$.




Two angles are complementary. One angle measures 37 degrees. Find the measure of the other angle.

Two angles are supplementary. One angle measures 127 degrees. Find the measure of the other angle.

## Student Response

1. 46 degrees
2. 39 degrees
3. 51 degrees
4. 48 degrees
5. 53 degrees

## Building on Student Thinking

If students struggle to see relationships of angles in figures, prompt students to look for complementary, supplementary, or vertical angles.

## Activity Synthesis

Ask students, "Were there any rows on which you and your partner did not get the same answer?" Invite students to share how they came to an agreement on the final answer for the problems in those rows.

Consider asking some of the following questions:

- "Did you and your partner use the same strategy for each row?"
- "What was the same and different about both of your strategies?"
- "Did you learn a new strategy from your partner?"
- "Did you try a new strategy while working on these questions?"

## Lesson Synthesis

Explain that a conjecture is a statement that we *think* is true but aren't certain about. It is more than just a guess. A conjecture could be a guess that is based on some evidence. Ask students to come up with a conjecture about vertical angles. Give students 1–2 minutes to draft a conjecture, then 1–2 minutes to share their conjecture with a partner. Invite a few students to share. (Vertical angles have the same measure)

Here are some additional questions that may be helpful to clarify their descriptions:

- What are vertical angles? (A pair of angles across from one another where two lines cross.)
- What is true about the measures of vertical angles? (The measures are always the same.)
- Do supplementary or complementary angles need to be next to one another? (No.) Think of examples where they are not.

Display diagrams and definitions of new vocabulary somewhere in the classroom so that students can refer back to them during subsequent lessons. "Vertical angles" is new vocabulary. You might consider also adding "intersecting lines" and "conjecture." As the unit progresses, new terms can be added.

## 3.5 Finding Angle Pairs

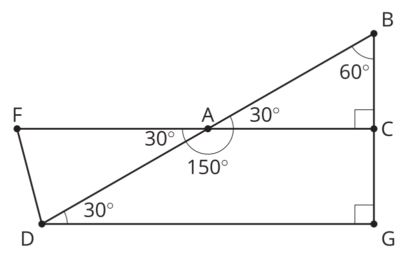
Cool-down

5 min

### Standards

Addressing **7.G.B.5**

### Student Task Statement




1. Name a pair of complementary angles in the diagram.
2. Name a pair of supplementary angles in the diagram.
3. Name a pair of vertical angles in the diagram.

### Student Response

1. $ABC$ with one of $BAC$, $FAD$, or $ADG$

2. One of these pairs:
    a. $CAD$ with one of $ABC$, $DAC$, or $DAG$
    b. $BAF$ with one of $ABC$, $DAC$, or $DAG$
    c. Any 2 of $BCA$, $ACG$, and $CGA$
3. One of these pairs:
    a. $DAF$ and $BAC$
    b. $BAF$ and $CAD$

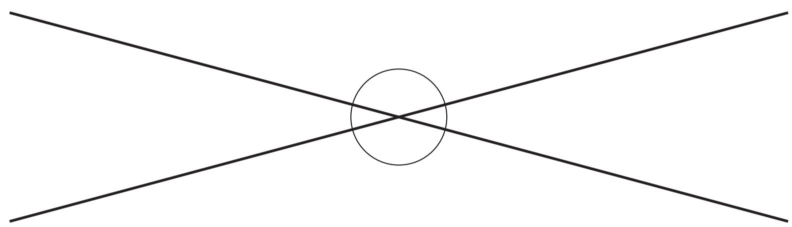
## Responding to Student Thinking

Points to Emphasize
If most students struggle with identifying types of angles, focus on angle relationships when doing this activity:

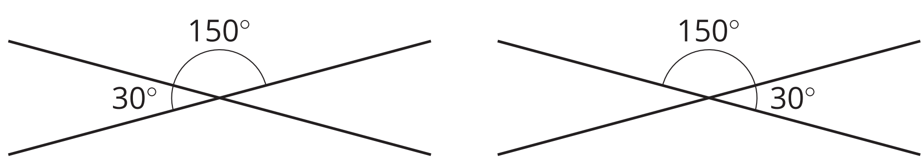
Grade 7, Unit 7, Lesson 4, Activity 2 Info Gap: Angle Finding

## Lesson 3 Summary

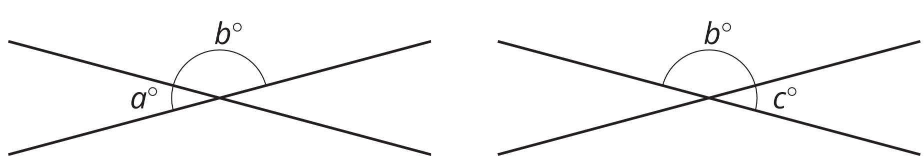
When two lines cross, they form two pairs of **vertical angles**. Vertical angles are across the intersection point from each other.



Vertical angles always have equal measure. We can see this because they are always supplementary with the same angle. For example:




This is always true!




$a + b = 180$ so $a = 180 - b$.

$c + b = 180$ so $c = 180 - b$.

That means $a = c$.

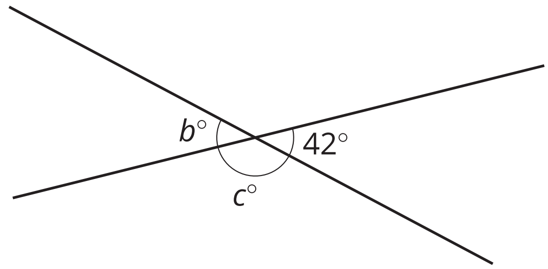
## Glossary

- vertical angles

# Lesson 3 Practice Problems

## 1 Student Task Statement

Two lines intersect. Find the value of $b$ and $c$.

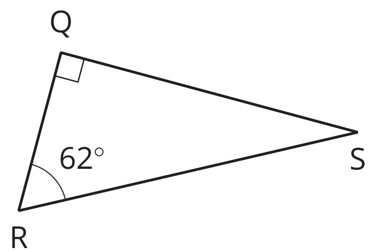



### Solution

$c = 138$, $b = 42$

## 2 Student Task Statement

In this figure, angles $R$ and $S$ are complementary. Find the measure of Angle $S$.




### Solution

$28^\circ$

## 3 Student Task Statement

If two angles are both vertical and supplementary, can we determine the angles? Is it possible to be both vertical and complementary? If so, can you determine the angles? Explain how you know.

### Solution

Yes, they are both possible. Vertical and supplementary angles must be $90^\circ$ each, because the two angles must be the same and add up to $180^\circ$. Vertical and complementary angles must be $45^\circ$, because the two angles must be the same and add up to $90^\circ$.

4 from Unit 6, Lesson 21

### Student Task Statement

Match each expression in the first list with an equivalent expression from the second list.

A. $5(x+1)-2x+11$

B. $2x+2+x+5$

C. $\frac{-3}{8}x-9+\frac{5}{8}x+1$

D. $2.06x-5.53+4.98-9.02$

E. $99x+44$

1. $\frac{1}{4}x-8$
2. $\frac{1}{2}(6x+14)$
3. $11(9x+4)$
4. $3x+16$
5. $2.06x+(-5.53)+4.98+(-9.02)$

### Solution

- A matches 4
- B matches 2
- C matches 1
- D matches 5
- E matches 3

5 from Unit 6, Lesson 19

### Student Task Statement

Factor each expression.

a. $15a-13a$

b. $-6x-18y$

c. $36abc+54ad$

### Solution

a. $a(15-13)$

b. $-6(1x+3y)$ (or $6(-x-3y)$)

c. $9a(4bc+6d)$

## 6 from Unit 6, Lesson 17

### Student Task Statement

The directors of a dance show expect many students to participate but don't yet know how many students will come. The directors need 7 students to work on the technical crew. The rest of the students work on dance routines in groups of 9. For the show to work, they need at least 6 full groups working on dance routines.

a. Write and solve an inequality to represent this situation, and graph the solution on a number line.

b. Write a sentence to the directors about the number of students they need.

### Solution

a. $\frac{x-7}{9} \geq 6, x \geq 61$. The number line should have a closed circle at $x = 61$. Some students may start at $x = 61$ and draw a line with an arrow extending to the right; others may draw dots on integers to the right of $x = 61$.

b. The directors need at least 61 students to show up. (Possibly, they may only be happy if they get 61, 70, 79, etc. students so they have even groups of nine.)

## 7 from Unit 2, Lesson 5

### Student Task Statement

A small dog gets fed $\frac{3}{4}$ cup of dog food twice a day. Using $d$ for the number of days and $f$ for the amount of food in cups, write an equation relating the variables. Use the equation to find how many days a large bag of dog food will last if it contains 210 cups of food.

### Solution

$f = 1.5d$ (or equivalent). The bag will last 140 days since $210 \div 1.5 = 140$.