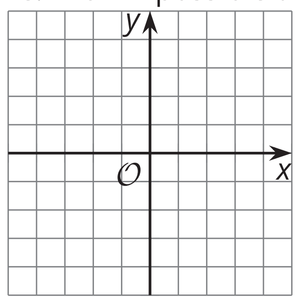
Unit 5, Lesson 5

Parallel Lines in the Plane

Let's investigate parallel lines in the coordinate plane.

5.1 Translating Lines

1. Draw any non-vertical line in the plane. Draw 2 possible translations of the line.

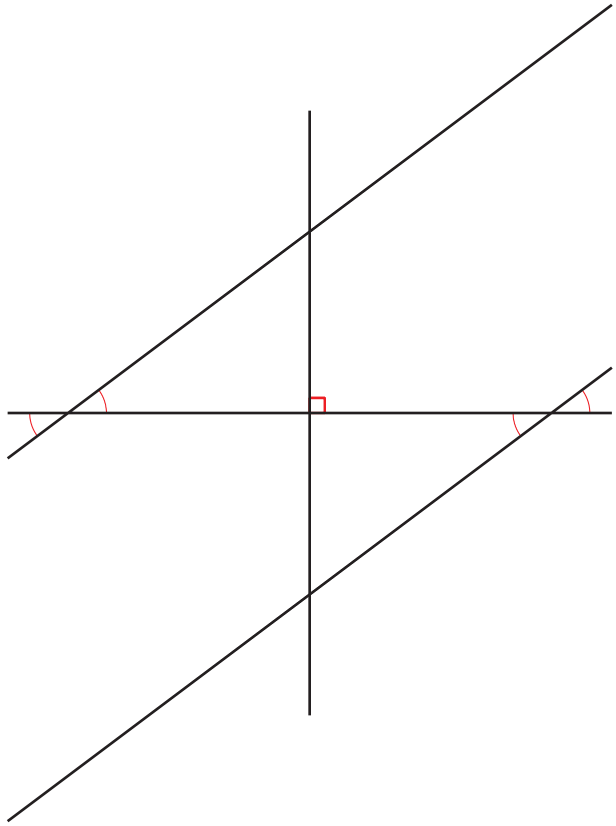


2. Find the slope of your original line and the slopes of the images.

5.2 Priya's Proof with Right Triangles

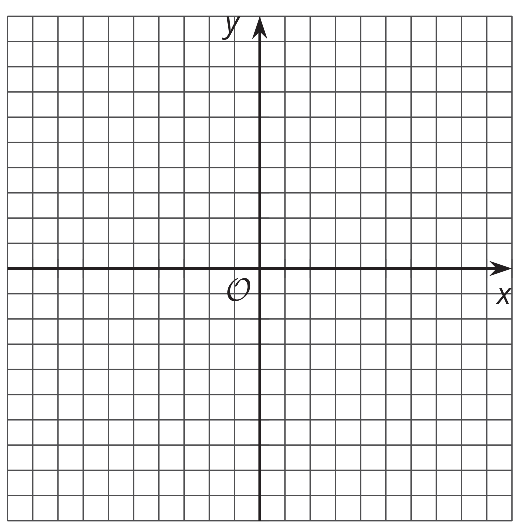
Priya writes a proof saying:

Consider any 2 parallel lines. Assume they are not horizontal or vertical. Construct a horizontal line that crosses both of the parallel lines. Then construct a vertical line through the midpoint of the horizontal segment between the lines. This forms 2 congruent triangles. Since the sides of the right triangles are horizontal and vertical, we can use them to find the slope of the parallel lines. Therefore the parallel lines have the same slope.



1. How does Priya know that the right triangles are congruent?
2. How does this prove that the slopes of parallel lines are equal?

5.3 Pair of Parallels



1. Graph the line $y = 2x + 3$. Label the point $(0, 3)$ on the graph as point A.

2. Draw the translation of the point $(0, 3)$ to $(-2, 5)$, including the directed line segment. Label the new point as point B.
3. Choose another point on the line, label it as point C, and draw the translation of that point by the same directed line segment. Label the new point as point D.
4. Find the equation of the line through B and D, then graph the line.
5. How do you know that AC is parallel to BD?
6. How do you know that AB is parallel to CD?

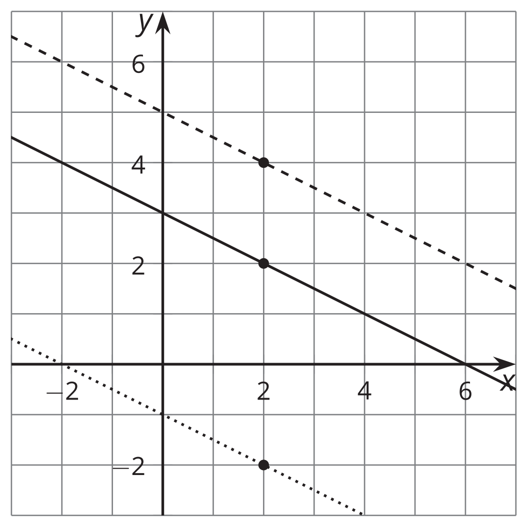
Are you ready for more?

Prove algebraically that the translation $(x, y) \rightarrow (x + p, y + q)$ takes a line to a line with the same slope. Consider starting with 2 points on the line with coordinates (x, y) and (w, z).

Lesson 5 Summary

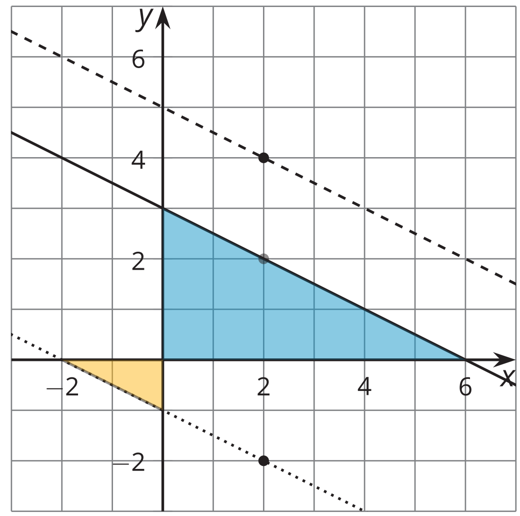
The solid line has been translated in 2 different ways:

1. by the directed line segment from $(2, 2)$ to $(2, 4)$ to produce the dashed line above
2. by the directed line segment from $(2, 2)$ to $(2, -2)$ to produce the dotted line below



The 3 lines look parallel to one another, as we would expect. We know that translations of lines result in parallel lines.

What happens to the slopes of these lines? If we draw in the slope triangles that go through the origin, we can see right triangles. Since we know the lines are parallel, the corresponding pairs of angles in the triangles must be congruent by the Alternate Interior Angles Theorem. Triangles with congruent angles are similar, and similar slope triangles result in lines with the same slope. Here we see slopes of $-\frac{5}{10}$, $-\frac{3}{6}$, and $-\frac{1}{2}$, which are all equal.



We can use similar reasoning to show that any 2 parallel lines that aren't vertical have the same slope, and also that any 2 lines with the same slope are parallel.

What if we wanted to find the equation of a line parallel to these 3 lines that goes through the point $(6, -1)$? We know the line must have the same slope of $-\frac{1}{2}$. We can use point-slope form and get $y + 1 = -\frac{1}{2}(x - 6)$.