Unit 8, Lesson 14

Infinite Decimal Expansions

Let's think about infinite decimals.

14.1 Searching for Digits

The first 3 digits after the decimal for the decimal expansion of $\frac{3}{7}$ have been calculated. Find the next 4 digits.

$$
\begin{array}{r|l}
 & 0.428 \\
\hline
7 & 3 \\
-\,2 & 8 \\
\hline
 & 20 \\
-\,1 & 4 \\
\hline
 & 60 \\
-\,5 & 6 \\
\hline
 & 4
\end{array}
$$

14.2 Some Numbers Are Rational

Your teacher will give your group a set of cards. Each card will have a calculations side and an explanation side.

1. The cards show Noah's work calculating the fraction representation of $0.4\overline{85}$. Arrange these in order to see how he figured out that $0.4\overline{85} = \frac{481}{990}$ without needing a calculator.
2. Use Noah's method to calculate the fraction representation of:

   a. $0.1\overline{86}$

   b. $0.7\overline{88}$

Are you ready for more?

Use this technique to find fractional representations for $0.\overline{3}$ and $0.\overline{9}$.

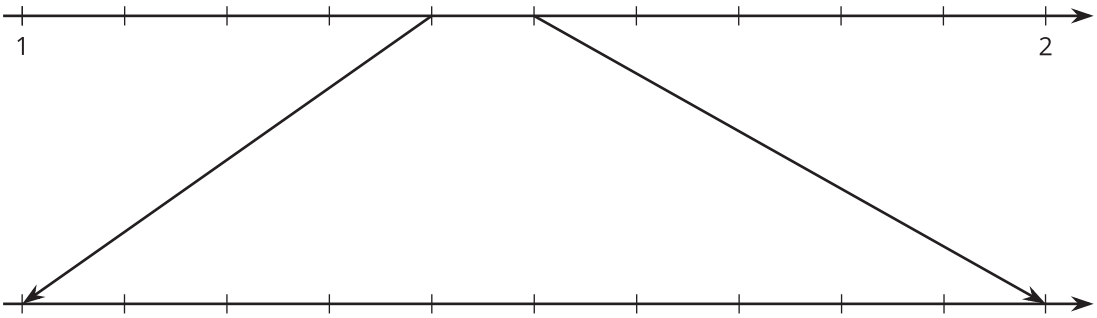
14.3 Some Numbers Are Not Rational

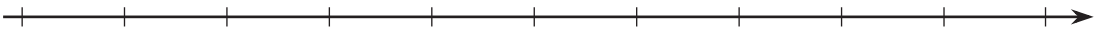
1. a. Why is $\sqrt{2}$ between 1 and 2 on the number line?

   b. Why is $\sqrt{2}$ between 1.4 and 1.5 on the number line?

   c. How can you figure out an approximation for $\sqrt{2}$ accurate to 3 decimal places?

   d. Label all of the tick marks. Plot $\sqrt{2}$ on all three number lines. Make sure to add arrows from the second to the third number line.

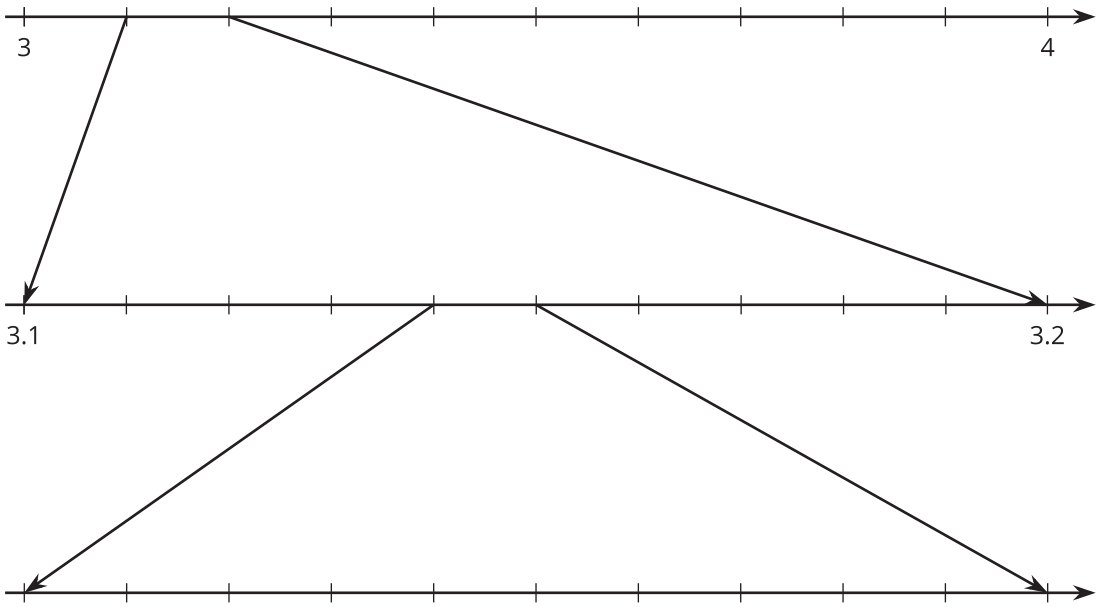




2. a. Elena notices that a beaker in science class says it has a diameter of 9 cm and measures its circumference to be 28.3 cm. Using these values and the equation for circumference, $C = 2\pi r$, what value do you get for π?

b. Diego learns that one of the space shuttle fuel tanks has a diameter of 840 cm and a circumference of 2,639 cm. Using these values and the equation for circumference, $C = 2\pi r$, what value do you get for π?

c. Label all of the tick marks on the number lines. Use a calculator to get a very accurate approximation of π and plot that number on all three number lines.



d. How can you explain the differences between these calculations of π?

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Lesson 14 Summary

Not every number is rational. Earlier we tried to find a fraction whose square is equal to 2. That turns out to be impossible, although we can get pretty close (try squaring $\frac{7}{5}$). Since there is no fraction equal to $\sqrt{2}$, it is not a rational number, so we call it an "irrational number." Another well-known irrational number is π.

Every number, rational or irrational, has a decimal expansion. For example, the rational number $\frac{2}{11}$ has the decimal expansion 0.181818 . . . with the 18s repeating forever. Irrational numbers also have infinite decimal expansions, but they don't end up having a repeating pattern.