



# Other Fractional Exponents

Let's look at powers of roots.

## 10.1 Breaking Up Fractions

1. Write these products as a single fraction.

a.  $3 \cdot \frac{1}{5}$

b.  $10 \cdot \frac{1}{9}$

c.  $\frac{1}{8} \cdot 6$

2. Write these fractions as the product of a whole number and a fraction with a numerator of 1.

a.  $\frac{2}{3}$

b.  $\frac{5}{4}$

c.  $1\frac{3}{5}$

## 10.2 Other Rational Exponents

Rewrite each expression using one or more roots (like  $\sqrt{2}$  or  $\sqrt[3]{2}$ ) and whole-number exponents.

1.  $\left(11^{\frac{1}{2}}\right)^3$

2.  $\left(b^3\right)^{\frac{1}{4}}$

3.  $3^{\frac{2}{5}}$

4.  $\left(a^2 \cdot b^4\right)^{\frac{1}{5}}$

5.  $\left(x^3 \cdot b^9\right)^{\frac{2}{3}}$

6.  $\left(y^{\frac{1}{2}} z^{\frac{3}{4}}\right)^8$



### Are you ready for more?

From the commutative property of multiplication,  $a \cdot \frac{1}{b} = \frac{1}{b} \cdot a$ .

1. Use exponents to show that  $(\sqrt[b]{x})^a = \sqrt[b]{x^a}$ .
2. Use technology and numbers for  $x$ ,  $a$ , and  $b$  to check that this is true, or describe patterns of numbers for when it is not true.

## 10.3 Half Days

Scientists are growing bacteria for an experiment. From past experiments, scientists estimate that the bacteria will grow in this environment so that the population  $t$  days after the experiment is started is  $300 \cdot 2^t$ .

1. What is the value of the expression when  $t = 0$ ? What does it mean in this situation?
2. What is the value of the expression when  $t = 1$ ? What does it mean in this situation?
3. What is the value of the expression when  $t = \frac{1}{2}$ ? What does it mean in this situation?

## Lesson 10 Summary

We can extend our connection between expressions with fractional exponents and expressions with roots to  $a^{\frac{b}{c}} = (\sqrt[c]{a})^b$ .

This means that we can interpret an expression like  $3^{\frac{2}{5}}$  as  $(\sqrt[5]{3})^2$  or as  $\sqrt[5]{3^2} = \sqrt[5]{9}$ .

Using the exponent rule can also be useful for writing roots in a simpler way. For example, it is true that  $\sqrt[4]{3^{10}} = \sqrt{3^5}$  because the first expression can be written as  $\sqrt[4]{3^{10}} = 3^{\frac{10}{4}}$ , which is equivalent to  $3^{\frac{5}{2}} = \sqrt{3^5}$ .

The rule can also be helpful when interpreting certain situations. For example, let's fill a chess board with rice by putting 1 grain of rice on the top left square, then doubling the amount of rice on each square as we go across the board. Will there ever be 1 million grains of rice on a square? If so, when?



We could write a function to represent the number of grains of rice on a square as  $R(s) = 2^s$ , where  $s$  represents the number of times the amount of rice on a square is doubled, so that there is 1 grain of rice to start, 2 grains after it has been doubled once, 4 grains after doubling twice, and so on. By graphing  $R(s)$  and  $y = 1,000,000$  we could figure out how many times it has been doubled to have 1 million grains of rice on a square. It turns out that the graphs intersect when  $s = 19.932$ . This means that when we double the rice on a square 20 times it will have more than 1 million grains of rice on it (and we are doubling the rice 43 more times for the last square of the board)!