Scope and Sequence for Grade 8

IM Grade 8 begins with transformational geometry. Students study rigid transformations and congruence, and then dilations and similarity. This provides background for understanding the slope of a line in the coordinate plane.

Next, students build on their understanding of proportional relationships, from IM Grade 7, to study linear relationships. They use equations, tables, and graphs to represent linear relationships, and make connections across these representations. Students expand their ability to work with linear equations in one and two variables, extending their understanding of a solution to an equation in one or two variables to comprehend a solution to a system of equations in two variables. They learn that linear relationships are an example of a special kind of relationship called a *function*. Students apply their understanding of linear relationships and functions to contexts involving data with variability.

The course ends the year with students extending their understanding of exponents to include all integers, and in the process codifying the properties of exponents. They learn about orders of magnitude and scientific notation in order to represent and compute with very large and very small quantities. They encounter irrational numbers for the first time and informally extend the rational-number system to the real-number system, motivated by their work with the Pythagorean Theorem.

The final unit of the course is optional. The lessons provide students with additional opportunities to integrate and apply various ideas from the course to solve real-world and mathematical problems.

Unit 1: Rigid Transformations and Congruence

In this unit, students explore translations, rotations, and reflections of plane figures in order to understand the structure of rigid transformations. They use the properties of rigid transformations to formally define what it means for shapes to be congruent.

In earlier grades, students studied geometric measurement to find angle measures and side lengths of two-dimensional figures as well as applied area and perimeter formulas for polygons including rectangles, parallelograms, and triangles. In this unit, students build on this work as they identify corresponding congruent angles and side lengths of figures and their images under rigid transformations. In an upcoming unit, students will explore dilations and similar figures in the plane.

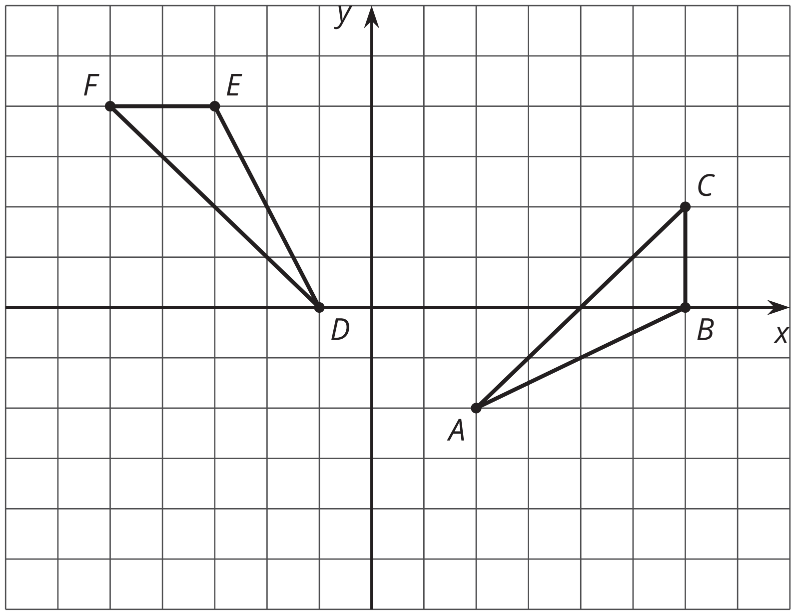
In the first section, students begin with an informal exploration of transformations in the plane, then increase their precision of language to describe translations, rotations, and reflections with formal descriptions, including coordinates (MP6).

Then students identify corresponding parts of figures and conclude that angles and distances are preserved under rigid transformations. Students use this property to reason about plane figures, including parallel lines cut by a transversal.

Students then learn the formal definition of "congruent" and use this definition to show that corresponding parts of congruent figures are also congruent. Finally, students apply their understanding of congruence and rigid motions to justify that the sum of the interior angles in a triangle must be 180°.

The lessons in this unit ask students to work on geometric figures that are not set in a real-world context. Students have opportunities to engage in real-world applications in the culminating lesson of the unit where they examine tessellations and other symmetric designs.

In this unit, students reason about congruence and justify properties of figures using rigid transformations, but they are not required to create a formal proof. They will prove these and other geometric properties more formally in later courses.



Progression of Disciplinary Language

In this unit, teachers can anticipate students using language for mathematical purposes, such as describing, generalizing, and justifying. Throughout the unit, students will benefit from routines designed to grow robust disciplinary language, both for their own sense-making and for building shared understanding with peers. Teachers can formatively assess how students are using language in these ways, particularly when students are using language to:

Describe

- Movements of figures (Lessons 1 and 2).
- Observations about transforming parallel lines (Lesson 9).
- Transformations using corresponding points, line segments, and angles (Lesson 10).
- Observations about angle measurements (Lesson 16).
- Transformations found in tessellations and in designs with rotational symmetry (Lesson 17).

Generalize

- About categories for movement (Lesson 2).
- About rotating line segments 180° (Lesson 8).
- About the relationship between vertical angles (Lesson 9).
- About transformations and congruence (Lesson 12).
- About corresponding segments and length (Lesson 13).
- About alternate interior angles (Lesson 14).
- About the sum of angles in a triangle (Lesson 16).

Justify

- Whether or not rigid transformations could produce an image (Lesson 7).
- Whether or not shapes are congruent (Lesson 11).
- Whether or not polygons are congruent (Lesson 12).
- Whether or not ovals are congruent (Lesson 13).

- Whether or not triangles can be created from given angle measurements (Lesson 15).

In addition, students are expected to explain and interpret directions for transforming figures and apply transformations to find specific images. Students are also asked to use language to compare rotations of a line segment and compare perimeters and areas of rectangles. Over the course of the unit, teachers can support students' mathematical understandings by amplifying (not simplifying) language used for all of these purposes as students demonstrate and develop ideas.

The table shows lessons where new terminology is first introduced in this course, including when students are expected to understand the word or phrase receptively and when students are expected to produce the word or phrase in their own speaking or writing. Terms that appear bolded are in the Glossary. Teachers should continue to support students' use of a new term in the lessons that follow where it was first introduced.

lesson	new terminology	
	receptive	productive
8.1.1	**vertex** plane measure direction	slide turn
8.1.2	**clockwise** **counterclockwise** **reflection** **rotation** **translation**	opposite
8.1.3	**image** angle of rotation center (of rotation) line of reflection	**vertex**
8.1.4	**transformation** **sequence of transformations** distance	**clockwise** **counterclockwise** reflect rotate translate
8.1.5	**coordinate plane** point segment coordinates x-axis y-axis	
8.1.6	polygon	angle of rotation center (of rotation) line of reflection
8.1.7	**rigid transformation** **corresponding** measurements preserve	**reflection** **rotation** **translation** measure point
8.1.8	midpoint	segment
8.1.9	**vertical angles** parallel intersect	distance

lesson	new terminology	
	receptive	productive
8.1.10		**image** **rigid transformation** midpoint parallel
8.1.11	**congruent** perimeter area	
8.1.12		**right angle** x-axis y-axis area
8.1.13		**corresponding**
8.1.14	**alternate interior angles** **transversal**	**vertical angles** **congruent** supplementary angles
8.1.15	**straight angle**	
8.1.16		**alternate interior angles** **transversal** **straight angle**
8.1.17	**tessellation** symmetry	

Section A: Rigid Transformations

- Lesson 1: Moving in the Plane
- Lesson 2: Naming the Moves
- Lesson 3: Grid Moves
- Lesson 4: Making the Moves
- Lesson 5: Coordinate Moves
- Lesson 6: Describing Transformations

Section B: Properties of Rigid Transformations

- Lesson 7: No Bending or Stretching
- Lesson 8: Rotation Patterns
- Lesson 9: Moves in Parallel
- Lesson 10: Composing Figures

Section C: Congruence

- Lesson 11: What Is the Same?
- Lesson 12: Congruent Polygons
- Lesson 13: Congruence

Section D: Angles in a Triangle

- Lesson 14: Alternate Interior Angles
- Lesson 15: Adding the Angles in a Triangle
- Lesson 16: Parallel Lines and the Angles in a Triangle

Section E: Let's Put It to Work

- Lesson 17: Rotate and Tessellate

Unit 2: Dilations, Similarity, and Introducing Slope

In this unit students learn what makes figures similar and justify claims of similarity. They are introduced to the slope of a line and use properties of similar triangles to write equations that can describe all points (x, y) on a given line.

In prior grades, students learned about the relationship between scale factors and scaled copies. Students expand on this in the first section where they learn about dilations as a new transformation that creates scaled copies.

In the next section, students connect dilations to earlier work with rigid transformations as they explain why two figures are similar by describing a sequence of translations, reflections, rotations, and dilations that take one figure to the other. They discover that angle measures in similar figures are preserved, which can be used to justify that two triangles are similar if they share two (or three) angle measures. Students also find that the quotients of corresponding side lengths in similar figures are equal. This along with the fact that side lengths in similar figures are all multiplied by the same scale factor allows students to calculate unknown lengths in similar figures.

In the following section, students use the similarity of slope triangles to understand why any two distinct points on a line determine the same slope. Using these same properties of similar triangles, students practice writing equations for a given line, though students are not expected at this time to write equations in the form $y = mx + b$.

The lessons in this unit ask students to work on geometric figures that are not set in a real-world context, as those tasks are sometimes contrived and hinder rather than help understanding. Students do have opportunities to tackle real-world applications in the culminating activity of the unit where students examine shadows cast by objects.

In this unit, several lesson plans suggest that each student have access to a *geometry toolkit*. Each toolkit contains tracing paper, graph paper, colored pencils, scissors, ruler, protractor, and an index card to use as a straightedge or to mark right angles, giving students opportunities to develop their abilities to select appropriate tools and use them strategically to solve problems (MP5). Note that even students in a digitally enhanced classroom should have access to such tools; apps and simulations should be considered additions to their toolkits, not replacements for physical tools.

Progression of Disciplinary Language

In this unit, teachers can anticipate students using language for mathematical purposes, such as describing, explaining, representing, and justifying. Throughout the unit, students will benefit from routines designed to grow robust disciplinary language, both for their own sense-making and for building shared understanding with peers. Teachers can formatively assess how students are using language in these ways, particularly when students are using language to:

Describe

- Observations about scaled rectangles (Lesson 1).

- Observations about dilated points, circles, and polygons (Lesson 2).
- Sequences of transformations (Lesson 6).
- Observations about side lengths in similar triangles (Lesson 9).

Explain

- How to apply dilations to find specific images (Lesson 5).
- How to determine whether triangles are congruent, similar, or neither (Lesson 8).
- Strategies for finding missing side lengths (Lesson 9).
- How to apply dilations to find specific images of points (Lesson 12).
- Reasoning for a conjecture (Lesson 13).

Represent

- Dilations using given scale factors and coordinates (Lesson 4).
- Figures using specific transformations (Lesson 6).
- Graphs of lines using equations (Lesson 12).

In addition, students are expected to use language to interpret directions for dilating figures and for creating triangles; compare dilated polygons and methods for determining similarity; critique reasoning about angles, sides, and similarity; justify whether polygons are similar; and generalize about points on a line and similar triangles.

The table shows lessons where new terminology is first introduced in this course, including when students are expected to understand the word or phrase receptively and when students are expected to produce the word or phrase in their own speaking or writing. Terms that appear bolded are in the Glossary. Teachers should continue to support students' use of a new term in the lessons that follow where it was first introduced.

lesson	new terminology	
	receptive	productive
8.2.1	scale factor scaled copy scaling	
8.2.2	**dilation** center of dilation dilate	
8.2.4		center of dilation scale factor
8.2.6	**similar**	dilate
8.2.7		**dilation**
8.2.9	quotient	
8.2.10		**slope** slope triangle
8.2.11	similarity x-coordinate y-coordinate equation of a line	quotient
8.2.13	estimate approximate / approximately	

Section A: Dilations

- Lesson 1: Projecting and Scaling
- Lesson 2: Circular Grid
- Lesson 3: Dilations with No Grid
- Lesson 4: Dilations on a Square Grid
- Lesson 5: More Dilations

Section B: Similarity

- Lesson 6: Similarity
- Lesson 7: Similar Polygons
- Lesson 8: Similar Triangles
- Lesson 9: Side Length Quotients in Similar Triangles

Section C: Slope

- Lesson 10: Meet Slope

- Lesson 11: Writing Equations for Lines
- Lesson 12: Using Equations for Lines

Section D: Let's Put It to Work

- Lesson 13: The Shadow Knows

Unit 3: Linear Relationships

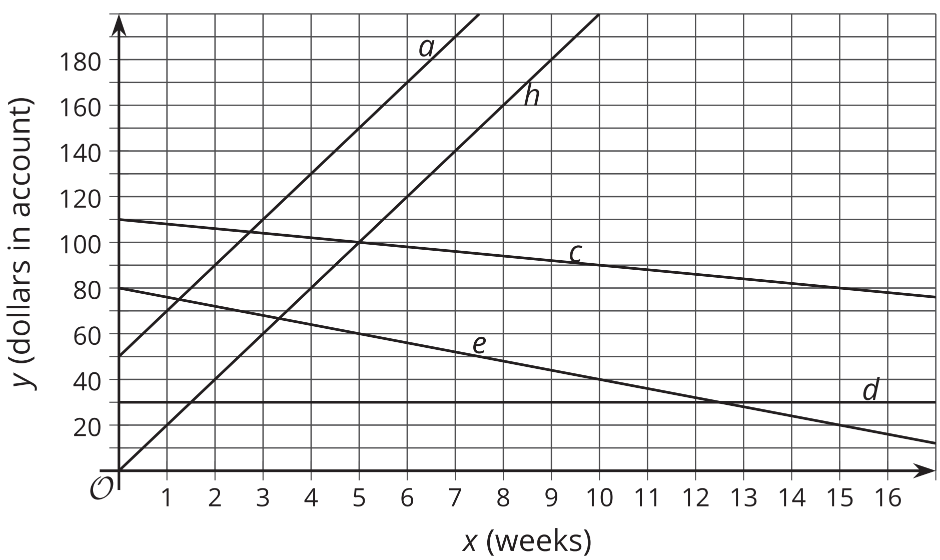
This unit introduces students to nonproportional linear relationships by building on earlier work with rates and proportional relationships from grade 7, and on earlier grade 8 work around similarity and slope.

The unit begins by revisiting different representations of proportional relationships. Students create graphs, tables, and equations in order to interpret the constant of proportionality in a context. They see the constant of proportionality between two variables as the rate of change of one variable with respect to the other.

Next, students analyze a relationship that is linear but not proportional. In this context, students see that the rate of change has a numerical value that is the same as the slope of the line that represents the relationship. Students also view the graph of a line in the coordinate plane as the vertical translation of a proportional relationship.

In the following section, students are introduced to lines with non-positive slopes and vertical intercepts. They consider situations represented by linear relationships with negative rates of change and establish a way to compute the slope of a line from any two distinct points on the line. Students also write equations of horizontal and vertical lines.

In the last section, students consider what it means for a pair of values to be a solution to an equation and the correspondence between coordinates of points on a graph and solutions of an equation.



Progression of Disciplinary Language

In this unit, teachers can anticipate students using language for mathematical purposes, such as representing, generalizing, and explaining. Throughout the unit, students will benefit from routines designed to grow robust disciplinary language, both for their own sense-making and for building shared understanding with peers. Teachers can formatively assess how students are using language in these ways, particularly when students are using language to:

Represent

- Situations involving proportional relationships (Lesson 1).
- Constants of proportionality in different ways (Lesson 3).

- Slope using expressions (Lesson 10).
- Linear relationships using graphs, tables, equations, and verbal descriptions (Lesson 5).
- Situations using negative slopes and slopes of zero (Lesson 9).
- Situations by graphing lines and writing equations (Lesson 13).
- Situations involving linear relationships (Lesson 15).

Generalize

- Categories for graphs (Lesson 2).
- About equations and linear relationships (Lesson 7).
- In order to make predictions about the slope of lines (Lesson 10).

Explain

- How to graph proportional relationships (Lesson 3).
- How to use a graph to determine information about a linear situation (Lessons 5 and 6).
- How to graph linear relationships (Lesson 10 and 11).
- How slope relates to changes in a situation (Lesson 11).

In addition, students are expected to describe observations about the equation of a translated line. Students will also have opportunities to use language to interpret situations involving proportional relationships, interpret graphs using different scales, interpret slopes and intercepts of linear graphs, justify reasoning about linear relationships, justify correspondences between different representations, and justify which equations correspond to graphs of horizontal and vertical lines.

The table shows lessons where new terminology is first introduced in this course, including when students are expected to understand the word or phrase receptively and when students are expected to produce the word or phrase in their own speaking or writing. Terms that appear bolded are in the Glossary. Teachers should continue to support students' use of a new term in the lessons that follow where it was first introduced.

lesson	new terminology	
	receptive	**productive**
8.3.1	represent scale label	constant of proportionality
8.3.2	equation	
8.3.3	**rate of change**	equation
8.3.5	**linear relationship** constant rate **rate of change**	slope
8.3.6	**vertical intercept** y-intercept	
8.3.7	initial (value or amount)	constant rate
8.3.8	relate	
8.3.9	horizontal intercept x-intercept	
8.3.10		**rate of change** **vertical intercept** y-intercept
8.3.12	constraint	horizontal line vertical line
8.3.13	**solution to an equation with two variables** variable combination set of solutions	

Section A: Proportional Relationships

- Lesson 1: Understanding Proportional Relationships
- Lesson 2: Graphs of Proportional Relationships
- Lesson 3: Representing Proportional Relationships
- Lesson 4: Comparing Proportional Relationships

Section B: Representing Linear Relationships

- Lesson 5: Introduction to Linear Relationships
- Lesson 6: More Linear Relationships
- Lesson 7: Representations of Linear Relationships
- Lesson 8: Translating to $y = mx + b$

Section C: Finding Slopes

- Lesson 9: Slopes Don't Have to Be Positive
- Lesson 10: Calculating Slope
- Lesson 11: Line Designs
- Lesson 12: Equations of All Kinds of Lines

Section D: Linear Equations

- Lesson 13: Solutions to Linear Equations
- Lesson 14: More Solutions to Linear Equations

Section E: Let's Put It to Work

- Lesson 15: Using Linear Relations to Solve Problems

Unit 4: Linear Equations and Linear Systems

In this unit, students work with writing equivalent equations and use reasoning to solve equations for a variable. Then students solve systems of linear equations using graphic and algebraic methods.

The unit begins with a focus on moves that can be done to write equivalent equations. At first, students use hanger diagrams as an intuitive representation of equality and represent their reasoning by labeling arrows that connect equivalent representations. With the reintroduction of negative values, students move away from hanger diagrams to algebraic equations and writing equivalent equations with the intention of solving for a variable.

Next, students examine the conditions under which equations could have 0, 1, or infinite solutions as a transition to thinking about similar situations involving systems of equations. Students finish the unit by examining systems of equations graphically and then finding solutions algebraically. They build on their understanding that the line representing an equation with 2 variables is made up of coordinate pairs that make the equation true. They find that the intersection of 2 lines is the point that makes both equations for the system true. Students also recognize when systems have no solution or infinite solutions based on the graphs and the slope and intercept.

Progression of Disciplinary Language

In this unit, teachers can anticipate students using language for mathematical purposes, such as critiquing, justifying, and generalizing. Throughout the unit, students will benefit from routines designed to grow robust disciplinary language, both for their own sense-making and for building shared understanding with peers. Teachers can formatively assess how students are using language in these ways, particularly when students are using language to:

Critique

- Strategies for writing equivalent equations (Lesson 1).
- Reasoning about maintaining balance in equations (Lesson 3).
- Solutions of linear equations (Lessons 4 and 5).
- Reasoning about structures of systems of equations (Lesson 14).
- Explanations of solutions (Lesson 16).

Justify

- Strategies for writing equivalent equations (Lessons 1 and 5).
- Predictions about maintaining balance (Lesson 2).
- Predictions about solutions of linear equations (Lesson 6).

Generalize

- About the structures of equations that have one, infinite, and no solutions (Lessons 7 and 8).
- About the structures of systems of equations (Lessons 14 and 15).

In addition, students are expected to use language to represent and interpret situations involving systems of linear equations, compare solutions of linear equations, and describe graphs of systems of linear equations.

The table shows lessons where new terminology is first introduced in this course, including when students are expected to understand the word or phrase receptively and when students are expected to produce the word or phrase in their own speaking or writing. Terms that appear bolded are in the Glossary. Teachers should continue to support students' use of a new term in the lessons that follow where it was first introduced.

lesson	new terminology	
	receptive	productive
8.4.1	representation equivalent expression	
8.4.2	expression	
8.4.3	solution to an equation distribute	
8.4.4	substitute	equation
8.4.5	term like terms distributive property factor	
8.4.6		term like terms distribute common denominator
8.4.7	no solution (only) one solution	
8.4.8	**constant term** coefficient linear equation infinitely many solutions	expression variable
8.4.11	ordered pair	
8.4.12	**system of equations** solution to a system of equations	
8.4.13	substitution	substitute no solution (only) one solution infinitely many solutions
8.4.14	algebraically	
8.4.15		**system of equations** substitution

Section A: Equivalent Equations

- Lesson 1: Writing Equivalent Equations
- Lesson 2: Keeping the Equation Balanced
- Lesson 3: Balanced Moves

- Lesson 4: More Balanced Moves
- Lesson 5: Solving Any Linear Equation
- Lesson 6: Strategic Solving

Section B: Linear Equations in One Variable

- Lesson 7: All, Some, or No Solutions
- Lesson 8: How Many Solutions?
- Lesson 9: When Are They the Same?

Section C: Systems of Linear Equations

- Lesson 10: On or Off the Line?
- Lesson 11: On Both of the Lines
- Lesson 12: Systems of Equations
- Lesson 13: Solving Systems of Equations
- Lesson 14: Solving More Systems
- Lesson 15: Writing Systems of Equations

Section D: Let's Put It to Work

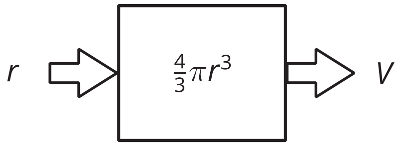
- Lesson 16: Solving Problems with Systems of Equations

Unit 5: Functions and Volume

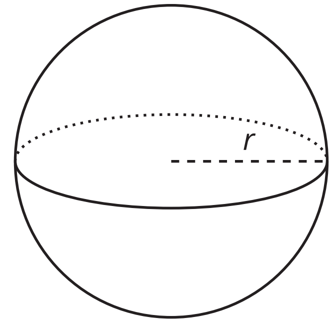
In this unit, students are introduced to the concept of a function as a relationship between "inputs" and "outputs" in which each allowable input determines exactly one output.

In the first three sections of the unit, students work with relationships that are familiar from previous grades or units (perimeter formulas, proportional relationships, linear relationships), expressing them as functions. They study the different ways functions can be represented, making connections between the representations and interpreting what they mean in context. Linear functions are a focus of the third section, and students will continue to work with linear functions in a later unit to model data. The use of function notation is left for a future course.

In the remaining three sections of the unit, students build on their knowledge of the formula for the volume of a right rectangular prism from grade 7, learning formulas for volumes of cylinders, cones, and spheres. Students express functional relationships described by these formulas as equations, focusing on situations involving proportional relationships. They use these relationships to reason about how the volume of a figure changes as one of its dimensions changes, transforming algebraic expressions to get the information they need. In future courses, students will continue this thinking as they study nonlinear relationships and question how, for example, the volume of a sphere changes as the radius increases.



r	V
0	0
2	$\frac{32}{3}\pi$
6	288π
r	$\frac{4}{3}\pi r^3$



Progression of Disciplinary Language

In this unit, teachers can anticipate students using language for mathematical purposes, such as generalizing, justifying, and comparing. Throughout the unit, students will benefit from routines designed to grow robust disciplinary language, both for their own sense-making and for building shared understanding with peers. Teachers can formatively assess how students are using language in these ways, particularly when students are using language to:

Generalize

- About what happens to inputs for each rule (Lesson 1).
- About dimensions of cylinders (Lesson 14).
- About the relationship between the volumes of cylinders and cones (Lesson 15).
- About dimensions of cones (Lesson 16).
- About volumes of spheres, cones, and cylinders as functions of their radii (Lesson 21).

Justify

- Claims about what can be determined from given information (Lesson 2).
- Claims about volumes of cubes and spheres based on graphs (Lesson 7).
- Claims about approximately linear relationships (Lesson 10).
- Reasoning about the volumes of spheres and cones (Lesson 21).

Compare

- Different representations of functions (Lesson 3).
- Features of graphs, equations, and situations (Lesson 4).
- Features of a situation with features of a graph (Lesson 6).
- Temperatures shown on a graph with different temperatures given in a table (Lesson 7).
- The volumes of cones with the volumes of cylinders (Lesson 16).
- Methods for finding and approximating the volume of a sphere as a function of its radius (Lesson 20).

In addition, students are expected to interpret representations of volume functions of cylinders, cones, and spheres and expected to describe the following: quantities in a situation, volume measurements and features of three-dimensional figures, the effects of varying dimensions of rectangular prisms and cones on their volumes, approximately linear relationships. Students are also expected to use language to represent relationships between volume and variable side length of a rectangular prism and relationships between volume and variable height of a cylinder, explain and represent how height and volume of cylinders are related, and explain reasoning about finding the volume of a cylinder and about the relationship between volumes of hemispheres and volumes of boxes, cylinders, and cones.

The table shows lessons where new terminology is first introduced in this course, including when students are expected to understand the word or phrase receptively and when students are expected to produce the word or phrase in their own speaking or writing. Terms that appear bolded are in the Glossary. Teachers should continue to support students' use of a new term in the lessons that follow where it was first introduced.

lesson	new terminology	
	receptive	productive
8.5.1	input output	
8.5.2	**function**	input output depends on
8.5.3	**independent variable** **dependent variable** radius	
8.5.5	prediction	
8.5.7	volume cube	
8.5.8	functional relationship linear function	**function**
8.5.9	mathematical model	prediction
8.5.10	piecewise linear function	linear function constant rate
8.5.11	cylinder three-dimensional	radius
8.5.12	cone sphere dimension	cylinder cube cubic centimeter rectangular prism
8.5.13	base (of a cylinder or cone) approximation for π	volume
8.5.14		base (of a cylinder or cone)
8.5.16		cone
8.5.19	hemisphere	
8.5.20		sphere
8.5.21	spherical	
8.5.22	approximate range	

Section A: Inputs and Outputs

- Lesson 1: Inputs and Outputs
- Lesson 2: Introduction to Functions

Section B: Representing and Interpreting Functions

- Lesson 3: Equations for Functions
- Lesson 4: Tables, Equations, and Graphs of Functions
- Lesson 5: More Graphs of Functions
- Lesson 6: Even More Graphs of Functions
- Lesson 7: Connecting Representations of Functions

Section C: Linear Functions and Rates of Change

- Lesson 8: Linear Functions
- Lesson 9: Linear Models
- Lesson 10: Piecewise Linear Functions

Section D: Cylinders and Cones

- Lesson 11: Filling Containers
- Lesson 12: How Much Will Fit?
- Lesson 13: The Volume of a Cylinder
- Lesson 14: Finding Cylinder Dimensions
- Lesson 15: The Volume of a Cone
- Lesson 16: Finding Cone Dimensions

Section E: Dimensions and Spheres

- Lesson 17: Scaling One Dimension
- Lesson 18: Scaling Two Dimensions
- Lesson 19: Estimating a Hemisphere
- Lesson 20: The Volume of a Sphere
- Lesson 21: Cylinders, Cones, and Spheres

Section F: Let's Put It to Work

- Lesson 22: Volume as a Function of . . .

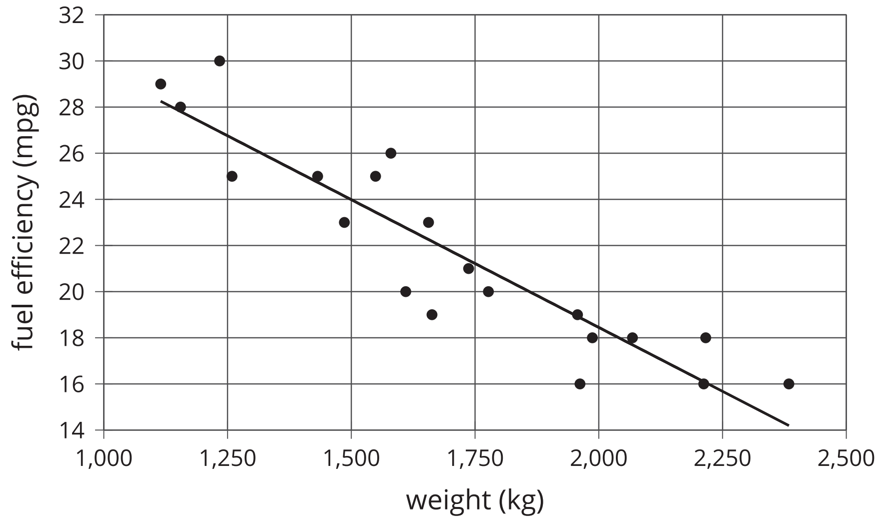
Unit 6: Associations in Data

In this unit, students analyze bivariate data. They will use scatter plots and fitted lines to analyze numerical data, and two-way tables, bar graphs, and segmented bar graphs to analyze categorical data. Students advance their understanding of lines by examining slopes in the context of data. They will revisit these data analysis topics in a later course in more depth. At this level, students should be able to construct and interpret points on a scatter plot, informally fit linear models to data, interpret a given linear model in the context of data, and generally recognize patterns of association using relative frequencies in a two-way table.

In prior grades, students analyzed data collected about one variable using dot plots, histograms, and box plots. This unit expands on that by considering the possible influence of a second variable on measurements about individuals.
In the first section, students are introduced to scatter plots and are reminded how to interpret points on a graph using a context. They also begin to recognize general trends in data.

In the second section, students look more closely at associations in data by informally drawing lines that model the general trend of the data. They also classify associations as positive, negative, linear, and non-linear by looking at the shape of the data in a scatter plot.

In the third section, students look at categorical data using two-way tables and relative frequencies. They then informally look at the relative frequencies to notice whether the variables are associated or not.



The unit ends with a lesson in which students collect and analyze numerical data using a scatter plot, then categorize the data based on a threshold and analyze the categories based on a two-way table.

Progression of Disciplinary Language

In this unit, teachers can anticipate students using language for mathematical purposes, such as explaining, representing, and interpreting. Throughout the unit, students will benefit from routines designed to grow robust disciplinary language, both for their own sense-making and for building shared understanding with peers. Teachers can formatively assess how students are using language in these ways, particularly when students are using language to:

Explain

- How to estimate using available data (Lesson 1).
- How to use tables and scatter plots to make estimates and predictions (Lesson 3).
- The meaning of slope for a situation (Lesson 6).
- How to use lines to show associations, identify outliers, and answer questions (Lesson 8).

Represent

- Data in organized ways (Lesson 1).
- Data using two-way tables, bar graphs, and segmented bar graphs (Lessons 9 and 10).
- Data using scatter plots (Lesson 11).

Interpret

- Situations and graphs involving bivariate data (Lesson 2).
- Tables and scatter plots of bivariate data (Lesson 3).
- Tables, scatter plots, equations, and situations involving bivariate data (Lesson 4).

In addition, students are expected to compare different representations of the same situation, describe and compare features of scatter plots, justify whether or not lines are good fits for a situation, and justify associations between bivariate data. Students also have opportunities to use language to generalize about what makes a line fit a data set well and about categories for sorting scatter plots.

The table shows lessons where new terminology is first introduced in this course, including when students are expected to understand the word or phrase receptively and when students are expected to produce the word or phrase in their own speaking or writing. Terms that appear bolded are in the Glossary. Teachers should continue to support students' use of a new term in the lessons that follow where it was first introduced.

lesson	new terminology	
	receptive	productive
8.6.1	**scatter plot**	
8.6.2	data display attribute	numerical data categorical data
8.6.4	**outlier** predict overpredict underpredict linear model	
8.6.5	**positive association** **negative association**	
8.6.6	linear association nonlinear association no association fitted line	
8.6.7	cluster	
8.6.8		**independent variable** **dependent variable** **positive association** **negative association** linear association
8.6.9	**segmented bar graph** **relative frequency** **two-way (frequency) table**	
8.6.11		**scatter plot** **outlier** cluster

Section A: Does This Predict That?

- Lesson 1: Organizing Data
- Lesson 2: Plotting Data

- Lesson 3: What a Point in a Scatter Plot Means

Section B: Associations in Numerical Data

- Lesson 4: Fitting a Line to Data
- Lesson 5: Describing Trends in Scatter Plots
- Lesson 6: The Slope of a Fitted Line
- Lesson 7: Observing More Patterns in Scatter Plots
- Lesson 8: Analyzing Bivariate Data

Section C: Associations in Categorical Data

- Lesson 9: Looking for Associations
- Lesson 10: Using Data Displays to Find Associations

Section D: Let's Put It to Work

- Lesson 11: Gone in 30 Seconds

Unit 7: Exponents and Scientific Notation

In this unit, students deepen their understanding of exponents, powers of 10, and place value before being introduced to scientific notation. They build on work done in a previous course where students focused on whole-number exponents with whole-number, fraction, decimal, or variable bases, but did not formulate rules regarding the use of exponents.

Students begin this unit by identifying patterns that emerge when multiplying and dividing powers of 10, and when raising powers of 10 to another power. Students generalize these patterns to develop exponent rules. They extend these rules to see why 10^0 must be equal to 1 and to understand what negative exponents mean.

Next, students determine that the rules developed for powers of 10 also work with other bases, as long as the bases in both expressions are the same. They observe a new rule that applies when multiplying bases that are different if the exponents are the same.

$a^n \cdot a^m = a^{n+m}$	$\left(a^n\right)^m = a^{n \cdot m}$
$\dfrac{a^n}{a^m} = a^{n-m}$	$a^0 = 1$
$a^{-n} = \dfrac{1}{a^n}$	$a^n \cdot b^n = (a \cdot b)^n$

In the next section, students return to working with powers of 10 as they use multiples of powers of 10 to describe magnitudes of very large and very small quantities, such as the distance from Earth to the sun in kilometers or the mass of a proton in grams. Students plot these large and small values on number lines labeled using exponents and see how these numbers can be expressed in different ways — for example as $75 \cdot 10^5$ or $7.5 \cdot 10^6$.

After building a foundation connecting powers of 10 with place value, students are finally introduced to scientific notation as a specific and useful way of writing numbers as a power of 10. They compute sums, differences, products, and quotients of numbers written in scientific notation to make additive and multiplicative comparisons, estimate quantities, and make measurement conversions.

Progression of Disciplinary Language

In this unit, teachers can anticipate students using language for mathematical purposes, such as critiquing, representing, and justifying. Throughout the unit, students will benefit from routines designed to grow robust disciplinary language, both for their own sense-making and for building shared understanding with peers. Teachers can formatively assess how students are using language in these ways, particularly when students are using language to:

Critique

- Reasoning about powers of powers (Lesson 3).
- Reasoning about zero exponents (Lesson 4).
- Applications of exponent rules (Lesson 7).
- Reasoning about scientific notation (Lesson 15).

Represent

- Situations using exponents (Lesson 1).
- Large and small numbers using number lines, exponents, and decimals (Lesson 9–11).
- Situations comparing quantities expressed in scientific notation (Lesson 14).

Justify

- Reasoning about multiplying powers of 10 (Lesson 2).
- Reasoning about powers of powers (Lesson 3).
- Reasoning about dividing powers of 10 (Lesson 4).
- Whether or not expressions are equivalent to exponential expressions (Lesson 6).
- Reasoning about situations comparing powers of 10 (Lesson 12).

In addition, students are expected to use language to generalize reasoning about repeated multiplication, generalize about patterns when multiplying different bases and exponents, describe how negative powers of 10 affect placement of decimals, and interpret situations comparing quantities expressed in scientific notation. Students also have opportunities to compare correspondences between exponential expressions and base-ten diagrams; compare expressions in scientific notation to other expressions; explain how to simplify expressions with negative powers of 10; and explain how to place and order large numbers on a number line.

The table shows lessons where new terminology is first introduced in this course, including when students are expected to understand the word or phrase receptively and when students are expected to produce the word or phrase in their own speaking or writing. Terms that appear bolded are in the Glossary. Teachers should continue to support students' use of a new term in the lessons that follow where it was first introduced.

lesson	new terminology	
	receptive	productive
8.7.1	**exponent** **base (of an exponent)** power factor reciprocal	repeated multiplication
8.7.2	powers of 10	
8.7.3	power of powers	
8.7.4	expanded positive exponent zero exponent	
8.7.5	negative exponent	positive exponent
8.7.6		**exponent** **base (of an exponent)** power zero exponent
8.7.7	evaluate	factor power of powers negative exponent
8.7.8	square (of a number)	
8.7.9	billion trillion multiple of	
8.7.10	integer	
8.7.12		multiple of
8.7.13	**scientific notation**	integer
8.7.14		powers of 10 billion trillion
8.7.15		**scientific notation**

Section A: Exponent Rules

- Lesson 1: Exponent Review
- Lesson 2: Multiplying Powers of 10
- Lesson 3: Powers of Powers of 10

- Lesson 4: Dividing Powers of 10
- Lesson 5: Negative Exponents with Powers of 10

Section B: More Exponent Rules

- Lesson 6: What about Other Bases?
- Lesson 7: Practice with Rational Bases
- Lesson 8: Combining Bases

Section C: Large and Small Numbers

- Lesson 9: Describing Large and Small Numbers Using Powers of 10
- Lesson 10: Representing Large Numbers on the Number Line
- Lesson 11: Representing Small Numbers on the Number Line
- Lesson 12: Applications of Arithmetic with Powers of 10

Section D: Scientific Notation

- Lesson 13: Definition of Scientific Notation
- Lesson 14: Estimating with Scientific Notation
- Lesson 15: Adding and Subtracting with Scientific Notation

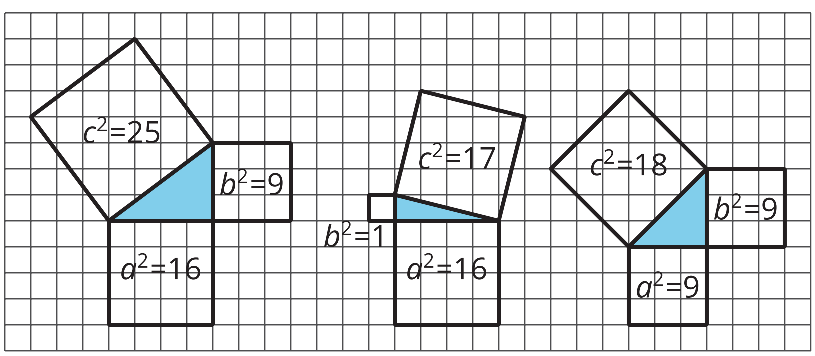
Section E: Let's Put It to Work

- Lesson 16: Is a Smartphone Smart Enough to Go to the Moon?

Unit 8: Pythagorean Theorem and Irrational Numbers

This unit introduces students to irrational numbers with a focus on connecting geometric and algebraic representations of square roots, cube roots, and the Pythagorean Theorem.

In the first section, students extend work from grade 6, composing and decomposing shapes to find the areas of tilted squares. They see "square root of n" and $\sqrt{n}$ to mean the side length of a square with area n square units, and understand that finding the solution to equations of the form $x^2 = n$ means determining which values of x make the equation true. Students learn and use definitions for "rational number" and "irrational number," learn (without proof) that $\sqrt{2}$ is irrational, and plot square roots on the number line.



In the second section, students continue using tilted squares as they investigate relationships between side lengths of right and non-right triangles. Students are encouraged to notice patterns among the triangles before being shown geometric and algebraic proofs of the Pythagorean Theorem. They use the Pythagorean Theorem and its converse to solve problems in two and three dimensions, for example, to determine lengths of diagonals of rectangles and right rectangular prisms, and to estimate distances between points in the coordinate plane.

In the third section, students see that "cube root of n" and $\sqrt[3]{n}$ mean the side length of a cube with volume n cubic units. They also represent a cube root as a decimal approximation and as a point on the number line.

In the fourth section, students consider the decimal expansions of rational and irrational numbers. They learn how to rewrite fractions as a repeating decimal, how to rewrite a repeating decimal as a fraction, and reinforce their understanding that irrational numbers have a place on the number line even if they cannot be written as a fraction of integers.

Progression of Disciplinary Language

In this unit, teachers can anticipate students using language for mathematical purposes, such as explaining, justifying, and comparing. Throughout the unit, students will benefit from routines designed to grow robust disciplinary language, both for their own sense-making and for building shared understanding with peers. Teachers can formatively assess how students are using language in these ways, particularly when students are using language to:

Explain

- Strategies for finding area (Lesson 1).
- Strategies for approximating and finding square roots (Lesson 5).
- Strategies for finding triangle side lengths (Lesson 7).
- Predictions about situations involving right triangles and strategies to verify (Lesson 11).
- Strategies for finding distances between points on a coordinate plane (Lesson 13).
- Strategies for approximating the value of cube roots (Lesson 15).

Justify

- Which squares have side lengths in a given range (Lesson 2).
- Ordering of irrational numbers (Lesson 6).
- Ordering of hypotenuse lengths (Lesson 10).

Compare

- Rational and irrational numbers (Lesson 4).
- Lengths of diagonals in rectangular prisms (Lesson 11).
- Strategies for approximating irrational numbers (Lesson 17).

In addition, students will do the following: Use language to generalize about area of squares, square roots, and approximations of side lengths and about the distance between any two coordinate pairs, critique reasoning about square root approximations and critique a strategy to represent repeating decimal expansions as fractions, describe observations about the relationships between triangle side lengths and describe hypotenuses and side lengths for given triangles, interpret diagrams involving squares and right triangles, interpret equations and approximations for the value of square and cube roots, and represent relationships between side lengths and areas.

The table shows lessons where new terminology is first introduced in this course, including when students are expected to understand the word or phrase receptively and when students are expected to produce the word or phrase in their own speaking or writing. Terms that appear bolded are in the Glossary. Teachers should continue to support students' use of a new term in the lessons that follow where it was first introduced.

lesson	new terminology	
	receptive	productive
8.8.2	**square root**	square (of a number)
8.8.4	**irrational number** square root symbol	**rational number**
8.8.5	diagonal decimal approximation	
8.8.6		**square root** square root symbol
8.8.7	**Pythagorean Theorem** **hypotenuse** **legs**	right triangle
8.8.10	converse of the Pythagorean Theorem	**Pythagorean Theorem**
8.8.11	edge length	**hypotenuse** **legs**
8.8.14	**cube root**	
8.8.15		**cube root** edge length
8.8.16	decimal representation finite decimal expansion	
8.8.17	**repeating decimal** infinite decimal expansion	**irrational number**

Section A: Side Lengths and Areas of Squares

- Lesson 1: The Areas of Squares
- Lesson 2: Side Lengths and Areas
- Lesson 3: Square Roots
- Lesson 4: Rational and Irrational Numbers
- Lesson 5: Square Roots on the Number Line
- Lesson 6: Reasoning about Square Roots

Section B: The Pythagorean Theorem

- Lesson 7: Finding Side Lengths of Triangles
- Lesson 8: A Proof of the Pythagorean Theorem
- Lesson 9: Finding Unknown Side Lengths
- Lesson 10: The Converse

- Lesson 11: Applications of the Pythagorean Theorem
- Lesson 12: More Applications of the Pythagorean Theorem
- Lesson 13: Finding Distances in the Coordinate Plane

Section C: Side Lengths and Volumes of Cubes

- Lesson 14: Edge Lengths and Volumes
- Lesson 15: Cube Roots

Section D: Decimal Representation of Rational and Irrational Numbers

- Lesson 16: Decimal Representations of Rational Numbers
- Lesson 17: Infinite Decimal Expansions

Section E: Let's Put It to Work

- Lesson 18: When Is the Same Size Not the Same Size?

Unit 9: Putting It All Together

In these optional lessons, students solve complex problems. In the first several lessons, they consider tessellations of the plane, understanding and using the terms "tessellation" and "regular tessellation" in their work, as well as using properties of shapes (for example, the sum of the interior angles of a quadrilateral is 360 degrees) and transformations to make inferences about regular tessellations. These lessons need to come after unit 8.1 has been done. In the later lessons, they investigate factors that impact predicting the temperature. In particular, they use scatter plots and lines of best fit to model the association between temperature and latitude. These lessons need to come after units 8.5 and 8.6 have been done.

All related standards in this unit have been addressed in prior units. These sections provide an *optional* opportunity for students to go more deeply and make connections between domains.

Progression of Disciplinary Language

In this unit, teachers can anticipate students using language for mathematical purposes, such as describing, representing, and justifying. Throughout the unit, students will benefit from routines designed to grow robust disciplinary language, both for their own sense-making and for building shared understanding with peers. Teachers can formatively assess how students are using language in these ways, particularly when students are using language to:

Describe

- Tessellations (Lesson 1)
- Associations in bivariate data (Lesson 5).

Represent

- The relationship between latitude and weather (Lesson 5).

Justify

- Claims about shapes that can and cannot be used to produce regular tessellations (Lesson 2).

The table shows lessons where new terminology is first introduced in this course, including when students are expected to understand the word or phrase receptively and when students are expected to produce the word or phrase in their own speaking or writing. Terms that appear bolded are in the Glossary. Teachers should continue to support students' use of a new term in the lessons that follow the one in which it was first introduced.

lesson	new terminology	
	receptive	productive
8.9.1		**tessellation** pattern
8.9.2	regular tessellation	regular polygon
8.9.6		mathematical model

Section A: Tessellations

- Lesson 1: Tessellations of the Plane
- Lesson 2: Regular Tessellations
- Lesson 3: Tessellating Polygons

Section B: Predicting the Temperature

- Lesson 4: What Influences Temperature?
- Lesson 5: Plotting the Temperature
- Lesson 6: Using and Interpreting a Mathematical Model

Pacing Guide

Number of days includes assessments. Upper bound of range includes optional lessons.

week 1, week 2, week 3, week 4, week 5, week 6, week 7, week 8, week 9, week 10, week 11, week 12, week 13, week 14, week 15, week 16, week 17, week 18, week 19, week 20, week 21, week 22, week 23, week 24, week 25, week 26, week 27, week 28, week 29, week 30, week 31, week 32, week 33, week 34, week 35

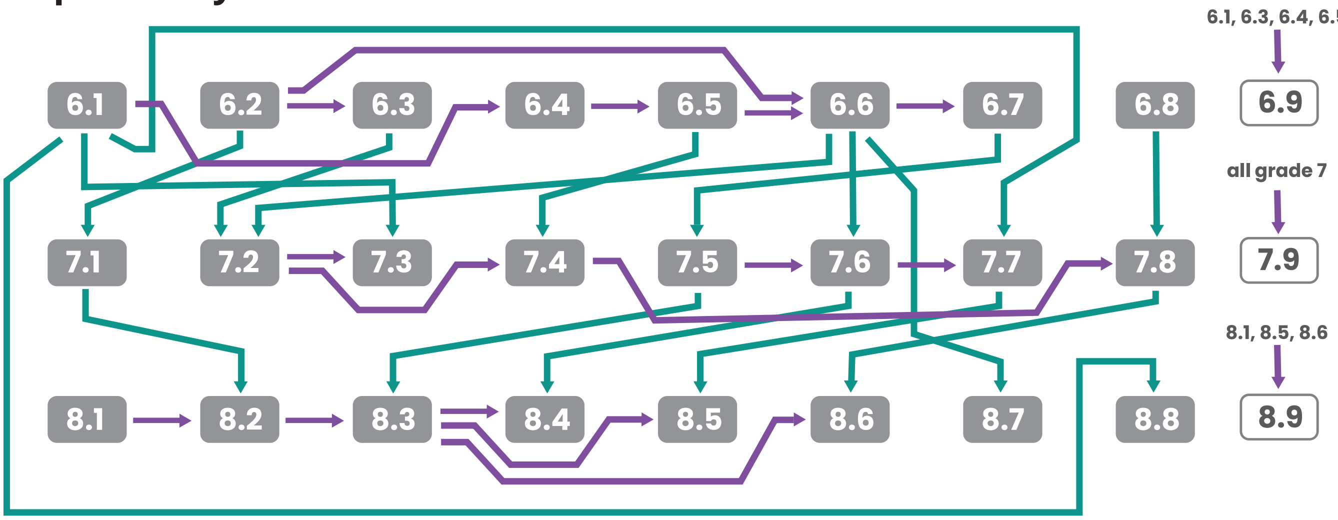
Grade 6	Grade 7	Grade 8
Unit 1 Area and Surface Area (20–22 days) (MA) Optional Lesson: 16, 19	Unit 1 Scale Drawings (12–15 days) Optional Lessons: 6, 8, 13	Unit 1 Rigid Transformations and Congruence (20 days) (MA) Optional Lessons: none
Unit 2 Introducing Ratios (19 days) Optional Lessons: none	Unit 2 Introducing Proportional Relationships (16–17 days) Optional Lessons: 14	Unit 2 Dilations, Similarity, and Introducing Slope (15 days) Optional Lessons: none
Unit 3 Unit Rates and Percentages (16–19 days) Optional Lesson: 2, 9, 16	Unit 3 Measuring Circles (11–13 days) Optional Lessons: 5, 11	Unit 3 Linear Relationships (16–17 days) Optional Lessons: 11
Unit 4 Dividing Fractions (16–20 days) (MA) Optional Lessons: 3, 4, 9, 16	Unit 4 Proportional Relationships and Percentages (17–18 days) Optional Lesson: 15	Unit 4 Linear Equations and Linear Systems (18 days) Optional Lessons: none
Unit 5 Arithmetic in Base Ten (15–18 days) (MA) Optional Lessons: 2, 9, 15	Unit 5 Rational Number Arithmetic (18–19 days) Optional Lesson: 10	Unit 5 Functions and Volume (23–25 days) (MA) Optional Lessons: 18, 22
Unit 6 Expressions and Equations (20–22 days) (MA) Optional Lessons: 11, 18	Unit 6 Expressions, Equations, and Inequalities (25 days) (MA) Optional Lessons: none	Unit 6 Associations in Data (12–13 days) Optional Lesson: 11
Unit 7 Rational Numbers (20–21 days) Optional Lesson: 19	Unit 7 Angles, Triangles, and Prisms (18–19 days) Optional Lesson: 17	Unit 7 Exponents and Scientific Notation (18 days) Optional Lessons: none
Unit 8 Data Sets and Distributions (20–21 days) (MA) Optional Lessons: 18	Unit 8 Probability and Sampling (21–23 days) (MA) Optional Lessons: 17, 20	Unit 8 Pythagorean Theorem and Irrational Numbers (20 days) Optional Lessons: none
Unit 9 Putting It All Together (0–11 days) Optional Lessons: all	Unit 9 Putting It All Together (0–12 days) Optional Lessons: all	Unit 9 Putting It All Together (0–6 days) Optional Lessons: all

(MA) = Unit has Mid-Unit Assessment

Total number of days for each course = Lessons + Assessments − Optional Lessons

Grade 6 = 146 Days Grade 7= 138 Days Grade 8 = 142 Days

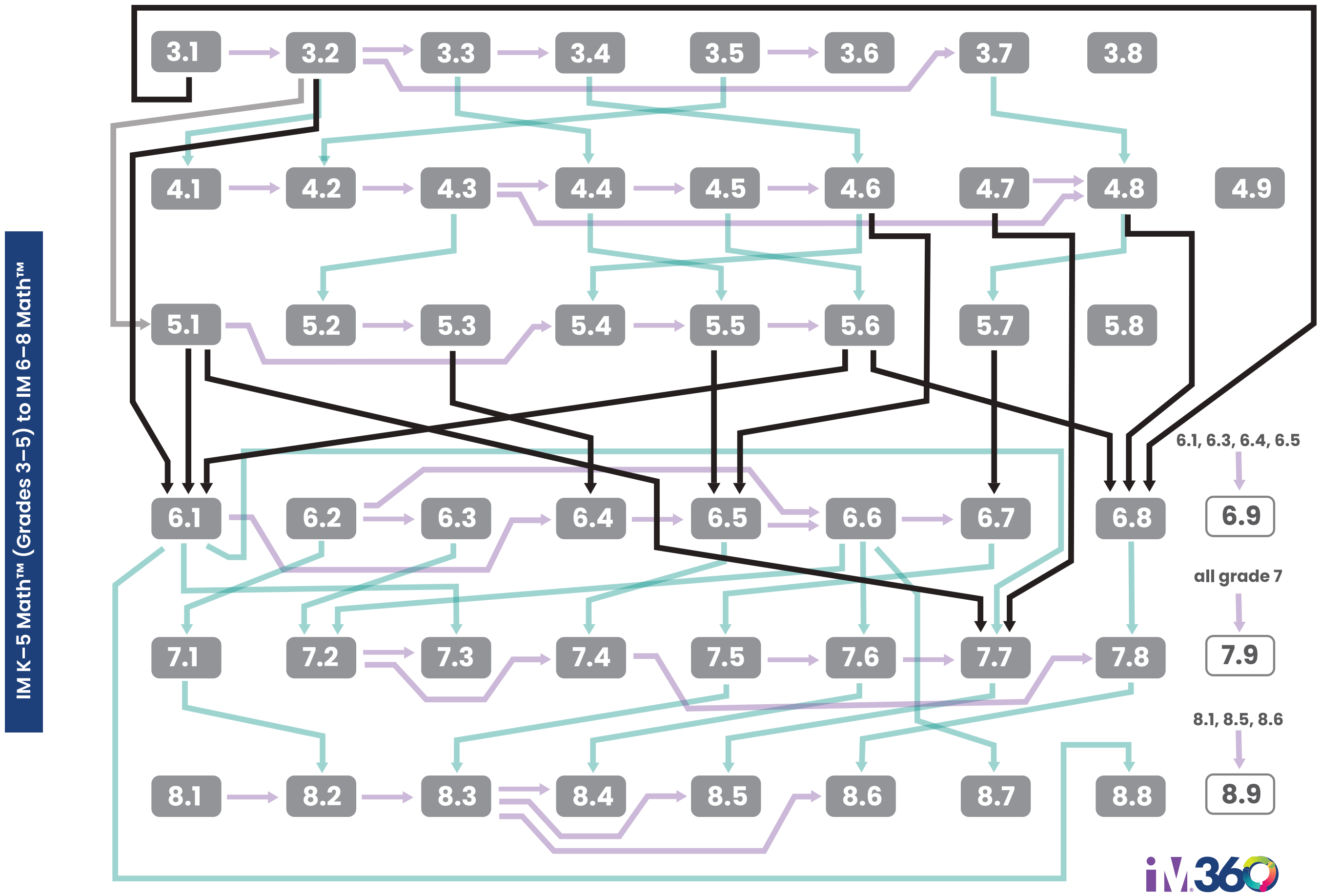
Dependency Chart



In the unit dependency chart, an arrow indicates that a particular unit is designed for students who already know the material in a previous unit. Reversing the order of the units would have a negative effect on mathematical or pedagogical coherence. Examples:

- There is an arrow from 6.2 to 6.6. Students are expected to use their knowledge of contexts involving ratios (from 6.2) to write and solve equations representing such contexts (in 6.6).
- There is an arrow from 7.4 to 7.8. Students are expected to use their skills in representing percentages (from 7.4) when solving problems about probability (in 7.8).
- There is an arrow from 8.3 to 8.6. Students are expected to use their skills in writing and interpreting an equation that represents a line (from 8.3) to interpret the parameters in an equation that represents a line that fits a scatter plot (in 8.6).

The following chart shows unit dependencies across the curriculum for IM Grades 3–8.


IM K–5 Math™ (Grades 3–5) to IM 6–8 Math™
3.1
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6.1, 6.3, 6.4, 6.5
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all grade 7
7.1
7.2
7.3
7.4
7.5
7.6
7.7
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7.9
8.1, 8.5, 8.6
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