Unit 3, Lesson 9

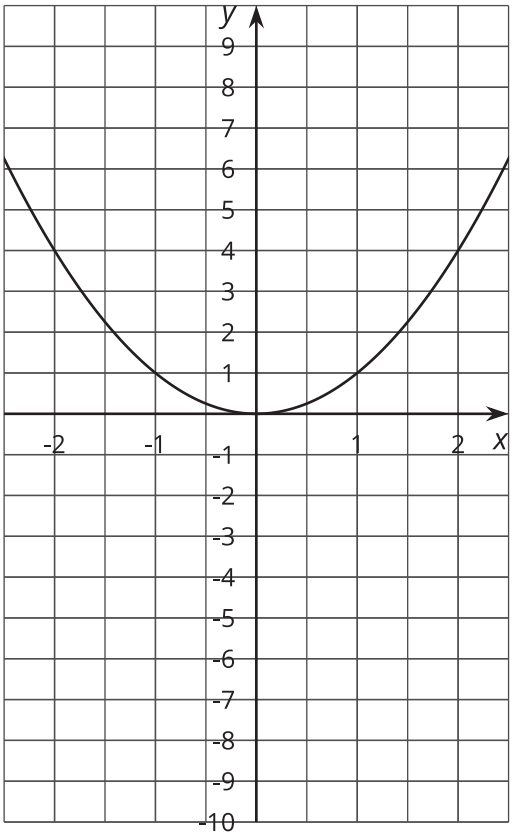
# Cubes and Cube Roots

Let's compare equations with cubes and cube roots.

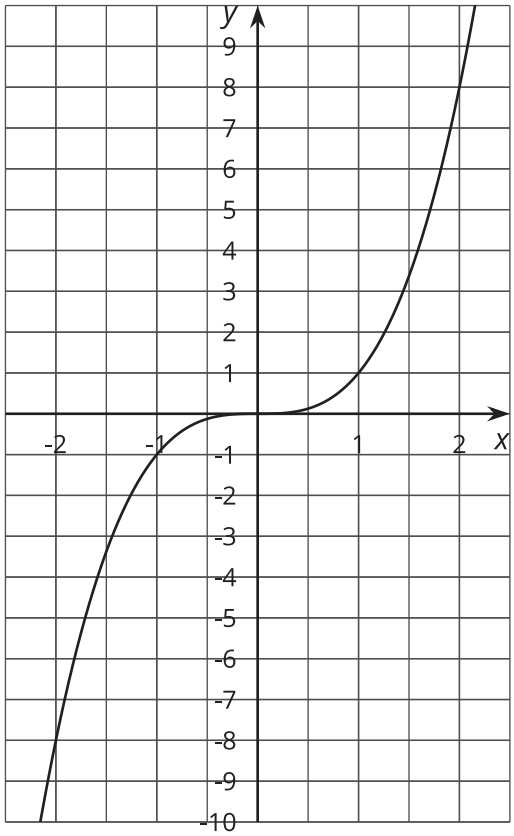
## 9.1 Comparing Two Graphs

How are these graphs the same? How are they different?

$y = x^2$



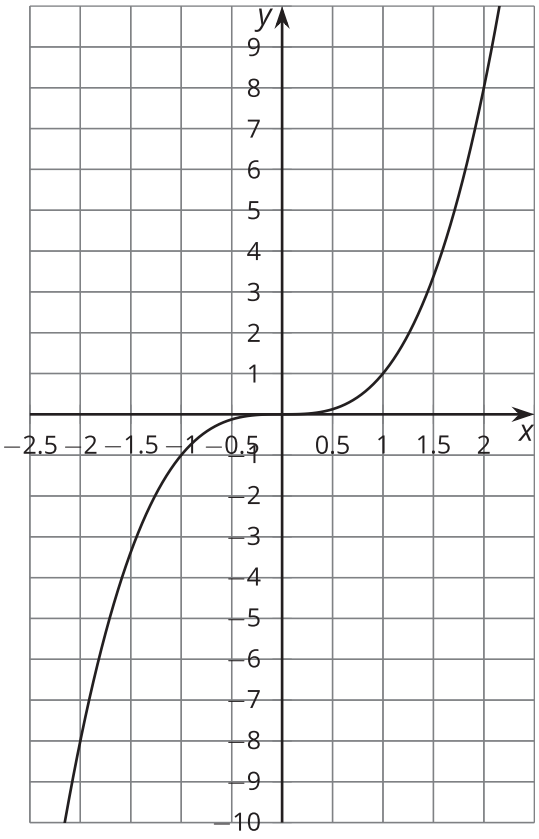

$y = x^3$



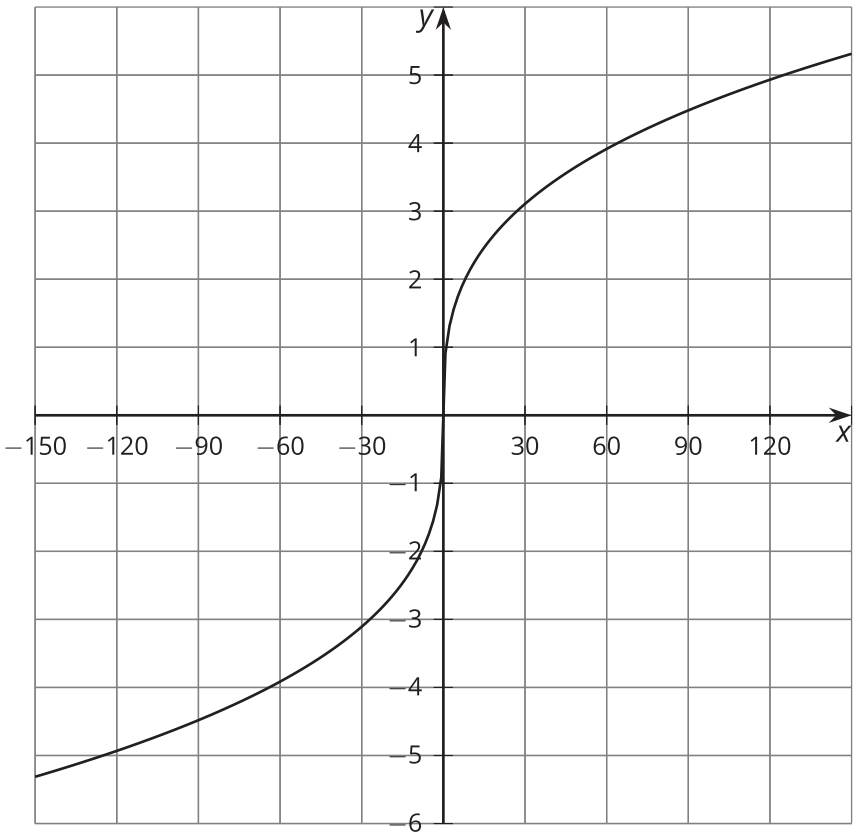
# 9.2 Finding Cube Roots with a Graph

How many real solutions are there to each of the following equations? Estimate the solution(s) from the graph of $y = x^3$. Check your estimate by substituting it back into the equation.

1. $x^3 = 8$
2. $x^3 = 2$
3. $x^3 = 0$
4. $x^3 = -8$
5. $x^3 = -2$



# 9.3 Cube Root Equations




1. Use the graph of $y = \sqrt[3]{x}$ to estimate the real solution(s) to $\sqrt[3]{x} = -4$.
2. Use the meaning of cube roots to find an exact real solution to the equation $\sqrt[3]{x} = -4$. How close was your estimate?

3. Find the real solution of the equation $\sqrt[3]{x} = 3.5$ using the meaning of cube roots. Use the graph to check that your solution is reasonable.

# 9.4 Solve These Equations with Cube Roots in Them

Here are a lot of equations:

- $\sqrt[3]{t+4} = 3$
- $-10 = -\sqrt[3]{a}$
- $\sqrt[3]{3-w} - 4 = 0$
- $\sqrt[3]{z} + 9 = 0$
- $\sqrt[3]{r^3 - 19} = 2$
- $5 - \sqrt[3]{k+1} = -1$
- $\sqrt[3]{p+4} - 2 = 1$
- $6 - \sqrt[3]{b} = 0$
- $\sqrt[3]{2n} + 3 = -5$
- $4 + \sqrt[3]{-m} + 4 = 6$
- $-\sqrt[3]{c} = 5$
- $\sqrt[3]{s-7} + 3 = 0$

1. Without solving, identify 3 equations that you think would be the least difficult to solve and 3 equations that you think would be the most difficult to solve. Be prepared to explain your reasoning.

2. Choose 4 equations and solve them. At least one should be from your "least difficult" list and at least one should be from your "most difficult" list.

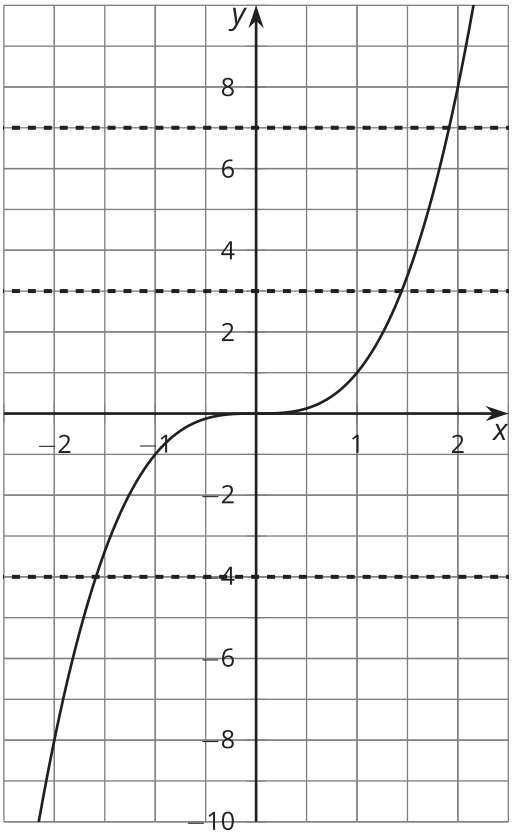
## Are you ready for more?

All of these equations are equivalent to an equation in the form $\sqrt[3]{ax+b}+c=0$ for some constants $a$, $b$, and $c$. Find a formula that would solve any such equation for $x$ in terms of $a$, $b$, and $c$.

## Lesson 9 Summary

Every real number has exactly one real cube root. You can see this by looking at the graph of $y=x^3$ on the real coordinate plane.

If $y$ is any number, for example, -4, then we can see that $y=$ -4 crosses the graph in one and only one place, so the equation $x^3=$ -4 will have the real solution -$\sqrt[3]{4}$. This is true for any real number $a$: $y=a$ will cross the graph in exactly one place, and $x^3=a$ will have one real solution, $\sqrt[3]{a}$.




In an equation like $\sqrt[3]{t}+6=0$, we can isolate the cube root, and then cube each side:

$$\begin{aligned}\sqrt[3]{t}+6&=0\\ \sqrt[3]{t}&=\text{-}6\\ t&=(\text{-}6)^3\\ t&=\text{-}216\end{aligned}$$

While cubing each side of an equation won't create an equation with solutions that are different from the original equation, it is still a good idea to always check solutions in the original equation because little mistakes can creep in along the way.