Trigonometry Squared

Let's connect the squares of some trigonometric ratios.

9.1 Squaring Sine, Cosine, and Tangent

1. Choose a number between 1 and 90. This number is your α.

2. Find the value of each expression. Round to the thousandths place.
   a. $\cos(\alpha)$

   b. $(\cos(\alpha))^2$

   c. $\sin(\alpha)$

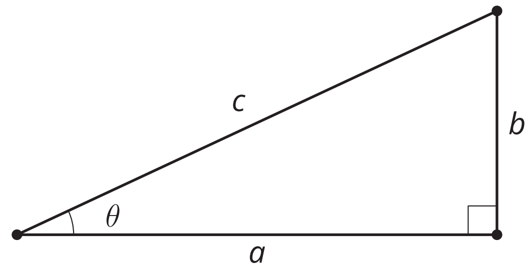
   d. $(\sin(\alpha))^2$

   e. $\tan(\alpha)$

   f. $(\tan(\alpha))^2$

9.2 Another Pythagorean Equation

Prove the conjecture that the class agreed upon. Use the labels provided in this right triangle for your proof.



Are you ready for more?

Prove $\tan(\beta) = \dfrac{\sin(\beta)}{\cos(\beta)}$.

9.3 Must It Be True?

Determine if each statement must be true, could possibly be true, or definitely can't be true. Explain or show your reasoning.

1. If $\sin(\theta) = \frac{3}{5}$, then the side opposite angle θ is 3.

2. If $\sin(\theta) = \frac{3}{5}$, then $\cos(\theta) = \frac{4}{5}$.

3. If the measure of angle α is 60 degrees, then $\cos(\alpha) = \frac{1}{2}$.

4. If $\cos(\beta) = \frac{1}{3}$, then $\sin(\beta) = \frac{2}{3}$.

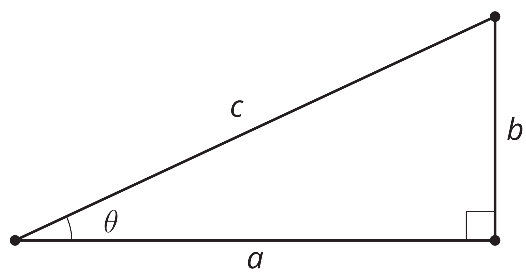
Lesson 9 Summary

In a previous lesson we saw a relationship between the cosine of an angle and the sine of its complementary angle. There is also a relationship between the cosine and sine of the same angle. Let's consider the squares of each value.

Here is a table with a few angle measures, the cosine and sine of each, and the squares of the cosine and sine of each.

angle	cosine	sine	square of cosine	square of sine
30°	0.866	0.5	0.75	0.25
35°	0.819	0.574	0.671	0.329
40°	0.766	0.643	0.587	0.413

We notice that the sum of the squares of the cosine of an angle and the sine of the same angle is always equal to 1. Summing squares reminds us of the Pythagorean Theorem, and this claim can be justified by using the Pythagorean Theorem.



In this right triangle, $a^2 + b^2 = c^2$ because of the Pythagorean Theorem. Divide both sides of the equation by c^2. Now the equation is $\frac{a^2}{c^2} + \frac{b^2}{c^2} = 1$. In this triangle, $\cos(\theta) = \frac{a}{c}$ by the definition of cosine, so $\cos^2(\theta) = \frac{a^2}{c^2}$. Similarly, $\sin^2(\theta) = \frac{b^2}{c^2}$. Substitution results in $\cos^2(\theta) + \sin^2(\theta) = 1$. This proof works for any right triangle, so the equation works for any acute angle θ.

We can use this equation to find the cosine of any acute angle when given the sine of the angle, and vice versa. For example, if we know the sine of an angle θ is $\frac{12}{13}$, we can substitute that value into the equation to get $\cos^2(\theta) + \frac{144}{169} = 1$. This is equivalent to $\cos(\theta) = \frac{25}{169}$. There are two solutions to this equation, one positive and one negative. Since the cosine of an acute angle is positive, we choose that value and get $\cos(\theta) = \frac{5}{13}$.