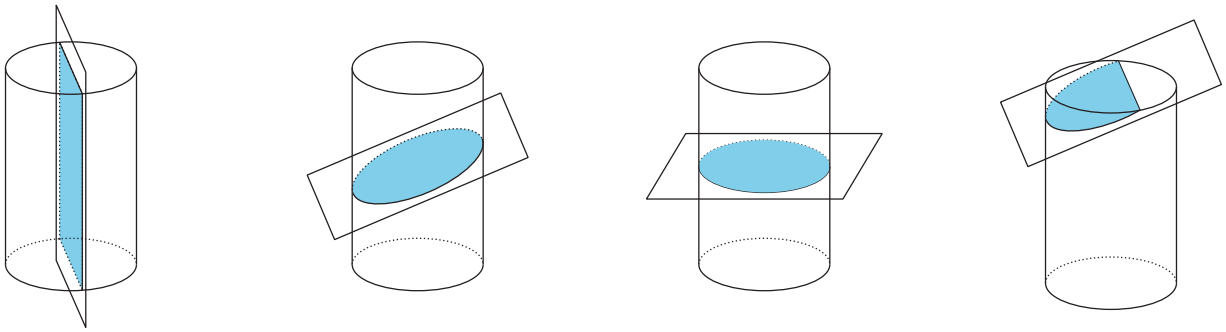
Unit 1 Family Support Materials

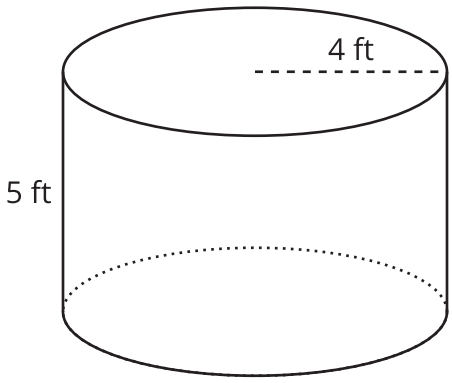
Solid Geometry

In this unit, your student will analyze properties of geometric solids. Because we live in three-dimensional space, people often need to solve problems about such solids. For example, a designer might need to create packaging for a candy bar in the shape of a triangular prism. An engineer might need to design a controller for a water tank in the shape of a cylinder. Or a lighting director for a theater might model the light from a spotlight, using the shape of a cone.

When working with solids, we often need to visualize cross-sections, or intersections between the solid and a plane. Here are all the kinds of cross-sections we can find in a cylinder.



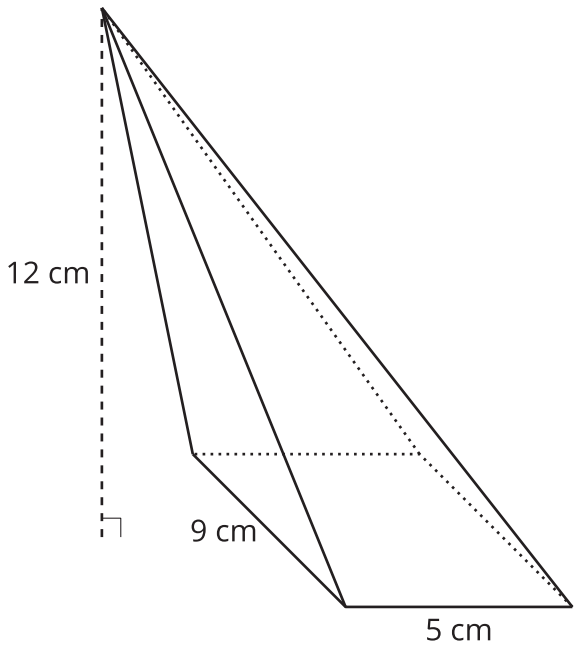
To find the volume of any prism or cylinder, no matter the shape of the base or if the figure is upright or oblique (slanted sideways), multiply the area of the base by the solid's height. This idea is captured in the formula $V = Bh$, where V is the volume, B is the area of the base, and h is the solid's height. For example, to find the volume of this cylinder, first calculate the area of the circular base using the expression πr^2, where r is the length of the base's radius. The base has an area of 16π square feet because $\pi(4)^2 = 16\pi$. So we can conclude that the volume of the cylinder is 80π cubic feet because $16\pi \cdot 5 = 80\pi$.



The process to find the volume of a pyramid or cone is the same as for prisms and cylinders,

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except that the result must be multiplied by $\frac{1}{3}$. That is, for pyramids and cones, $V = \frac{1}{3}Bh$.



For example, to find the volume of this rectangular pyramid, start by calculating the area of the base, which is 45 square centimeters because $5 \cdot 9 = 45$. When we substitute 45 and 12 into the volume formula, we find that the volume of the pyramid is 180 cubic centimeters:

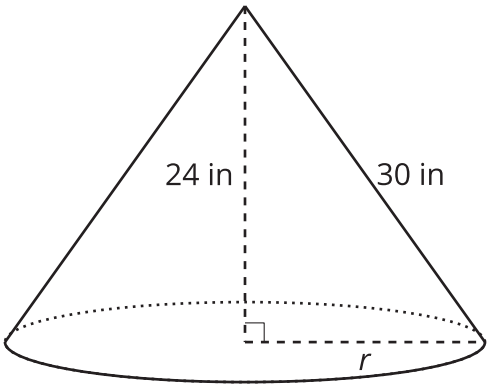
$V = \frac{1}{3}Bh$

$V = \frac{1}{3} \cdot 45 \cdot 12$

$V = 180$

Here is a task to try with your student:

Here is a cone.



1. One measurement that you need to calculate the volume is missing. Find the value of this

measurement.

2. Calculate the volume of the solid.

Solution:

1. The length of the radius is missing. Because this is a right triangle, the Pythagorean Theorem applies. One of the triangle's legs measures 24 inches, and the hypotenuse measures 30 inches, so $24^2 + r^2 = 30^2$. Squaring the 24 and the 30, we get $576 + r^2 = 900$. Subtract 576 from both sides to get $r^2 = 324$. Because r is the positive number that squares to get 324, we know that $\sqrt{324} = 18$, so the radius measures 18 inches.

2. The formula for the volume of a cone is $V = \frac{1}{3}Bh$. The cone's base is a circle with a radius of 18 inches. The area of the base is 324π square inches because $\pi(18)^2 = 324\pi$. When we substitute this area and the cone's height of 24 inches into the volume formula, we find that the volume of the cone is $2{,}592\pi$ cubic inches:

   $V = \frac{1}{3}Bh$

   $V = \frac{1}{3} \cdot 324\pi \cdot 24$

   $V = 2{,}592\pi$