Unit 2 Family Support Materials

Polynomial Functions

In this unit, your student will learn about a kind of function called a *polynomial*. A polynomial is a sum of terms involving only one letter, called a variable, and in which the exponents of the variable are whole numbers. For example, $3x^3 - x^2 + 10$ and $5x^6$ are polynomials. But $6x^2 + 2x^{-1}$ is not, because one of the exponents is negative. And $2xy - 7x$ is not, because it involves more than one variable. Your student will represent polynomial functions with graphs and equations, as they did in a previous course when they worked with two special kinds of polynomials—linear and quadratic functions.

Multiplication and division of numbers will be extended to polynomials, so this is a good time to refresh skills with multiplying and dividing numbers by hand. When numbers are multiplied, we often use the distributive property, in which each piece of one number is multiplied by each piece of the other number. For example, 34 is 30 plus 4, or 3 tens plus 4 ones. The tens and ones of each number are multiplied by the tens and ones of the other, and then all the results are added. When polynomials are multiplied, we also use the distributive property. Here is an example of each:

$$
\begin{aligned}
&(30 + 4)(10 + 5)\\
&= 30(10 + 5) + 4(10 + 5)\\
&= 30 \cdot 10 + 30 \cdot 5 + 4 \cdot 10 + 4 \cdot 5\\
&= 300 + 150 + 40 + 20\\
&= 510
\end{aligned}
$$

$$
\begin{aligned}
&(x - 7)(2x + 3)\\
&= x(2x + 3) + (\text{-}7)(2x + 3)\\
&= x \cdot 2x + x \cdot 3 + (\text{-}7) \cdot 2x + (\text{-}7) \cdot 3\\
&= 2x^2 + 3x - 14x - 21\\
&= 2x^2 - 11x - 21
\end{aligned}
$$

Multiplication, with numbers or polynomials, can be represented in a lot of ways, and your student should find a way that makes sense and is useful. Ask your student to show you how to multiply polynomials.

Long division with polynomials looks a lot like long division with numbers. Here is an example of each:

$$
\begin{array}{r}
31\\
12\overline{)372}\\
\underline{-36}\\
12\\
\underline{-12}\\
0
\end{array}
$$

$$
\begin{array}{r}
3x + 1\\
x + 2\overline{)3x^2 + 7x + 2}\\
\underline{-3x^2 - 6x}\\
x + 2\\
\underline{-x - 2}\\
0
\end{array}
$$

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Division can also be represented in many ways, so if you or your student learned a different way of doing long division, that way can also be extended to polynomials.

Here are some tasks to try with your student:

1. Multiply 47 by 25, using any method you like. Try using that same method to multiply $(4x + 7)(2x + 5)$. What was the same? What was different?
2. Divide 372 by 12, using any method you like. Then represent the division another way—for example, using pictures or words.
3. Factor these expressions. Check your answers by multiplying the factors. When you were factoring and multiplying, how did you know what to do at each step?
   a. $x^2 + 5x + 6$
   b. $x^2 + 2x - 8$

Solution:

1. One way to multiply 47 by 25 is to use a standard multiplication algorithm. We can also do something similar with $(4x + 7)(2x + 5)$. Just as we multiplied 47 by 5 and then by 20 and then added the results together, we can multiply $4x + 7$ by 5 and then by $2x$ and then add the results together. Here are two versions:

$$\begin{array}{r} 47 \\ \times 25 \\ \hline 235 \\ +940 \\ \hline 1175 \end{array} \qquad \begin{array}{r} 4x + 7 \\ \times\ 2x + 5 \\ \hline 20x + 35 \\ +\ 8x^2 + 14x + \ \ 0 \\ \hline 8x^2 + 34x + 35 \end{array}$$

2. One way to divide 372 by 12 is the standard division algorithm (shown earlier). Another way to do it is by subtraction. To be more efficient, we can take away groups of 120 (ten 12s) until the result is less than 120, and then take away groups of 12. We can take away 3 groups of 120 and 1 group of 12 from 372, and then we have nothing left over. So there are 31 groups of 12.

3. a. $x^2 + 5x + 6 = (x + 3)(x + 2)$

   b. $x^2 + 2x - 8 = (x + 4)(x - 2)$
   When factoring the first expression, I thought of two numbers whose product was 6 and sum was 5, and found 2 and 3. Similarly, for the second expression, the product of 4 and -2 is -8, and the sum of 4 and -2 is 2.

   When multiplying the factors to check my answer, I knew that I had to multiply each piece of the first factor by each piece of the second factor.