Unit 3 Family Support Materials

Rationals, Radicals, and Identities

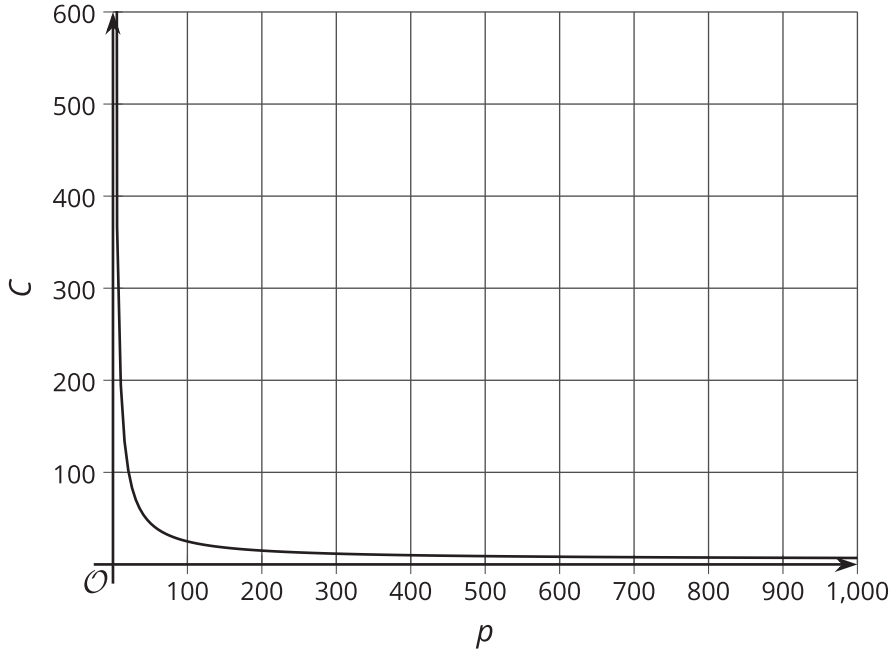
In this unit, your student will learn about rational functions. A rational function is a fraction with a polynomial in the numerator and denominator. Earlier, students worked with polynomial functions, which are a type of rational function with a denominator of 1.

We can see an example of a rational function when determining average cost. Say a school's Volunteer Club is hosting an annual family fun night to kick off the year and raise some money through ticket sales and donations to support their work helping the community. To encourage people to come, they plan to have inflatables and party games, which cost $2,000 to rent. The club will spend $5 per person for snacks and prizes. This means the average cost C per person p is

$$C(p) = \frac{2000+5p}{p}$$

At one extreme, if only 1 person comes, then the average cost per person is $2,005. If 100 people show up, the average cost per person is $25. If 200 people show up, the average cost per person is $15. If 1,000 people show up, the average cost per person is just $7.

The graph of the function C shown here is one example of what a rational function could look like. The value of C is very high for numbers that are close to 0, decreases quickly as the value of p increases, and then almost seems to level out as the value of p gets higher.

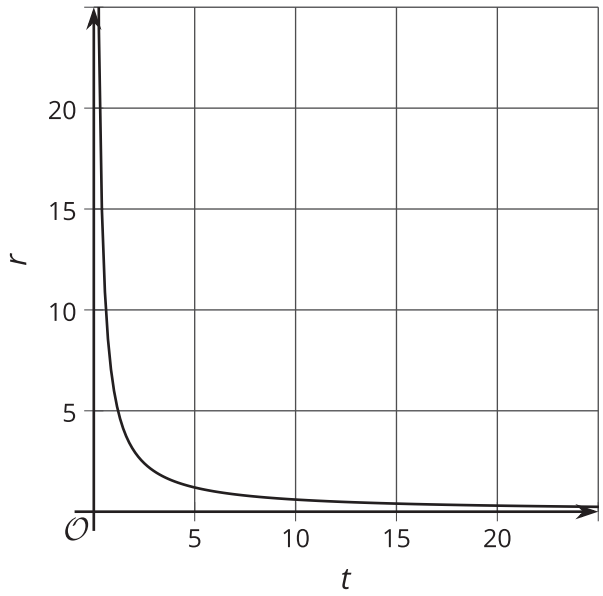


Based on last year's numbers, the club expects about 350 people to show up, which puts the cost per person at a little under $11. If the club charges $15 per ticket, then they'll raise about $1,400

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just from ticket sales.

Here is a task to try with your student:



Here is a graph showing the time t it would take to bike 6 miles for different rates r. Since $\text{distance} = \text{rate} \cdot \text{time}$, we can also say that $\text{time} = \text{distance} \div \text{rate}$, or $t = \frac{d}{r}$. In this case, $d = 6$, so this is the graph of $t = \frac{6}{r}$.

1. How long will it take to bike the distance at an average rate of 8 miles per hour?
2. What average rate is needed to finish 6 miles in 24 minutes?
3. As r gets closer and closer to 0, what does the graph tell you about the situation? What happens when $r = 0$?

Solution:

1. 45 minutes (or 0.75 hour)
2. 15 miles per hour (or 0.25 mile per minute)
3. For slower and slower rates, the time it takes to bike 6 miles increases. $r = 0$ doesn't make sense because you'll never travel 6 miles if you have no speed.