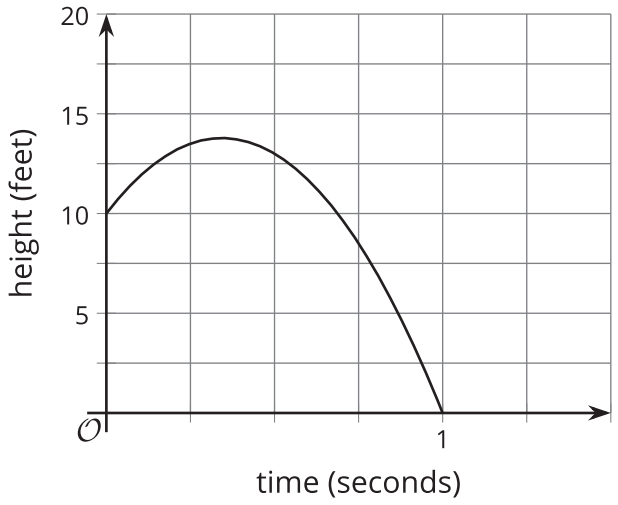
Unit 5 Family Support Materials

Transformations of Functions

In this unit, your student will move graphs of functions around the plane and figure out how to write new functions representing these graphs. Many professions use functions to model real-world relationships. For example, an economist might study the relationship between price and revenue. An engineer might study the relationship between temperature and efficiency of an engine. A psychologist might study the relationship between screen time and anxiety. Analyzing changes to a graph representing a relationship can help people understand changes in the real-world relationship that is being modeled.

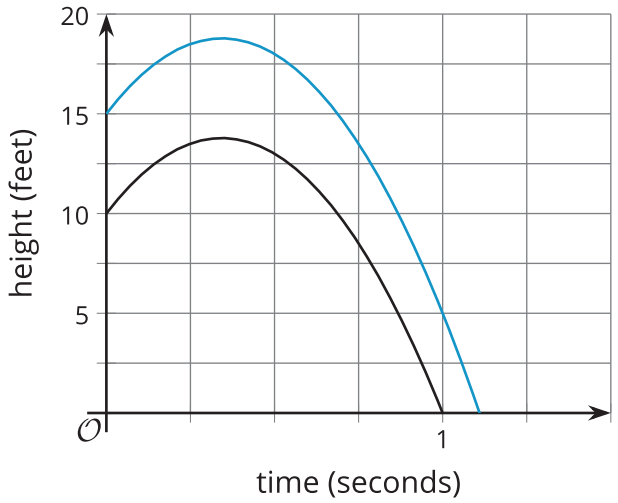
For example, here is a graph representing the height of a diver above the water after jumping from a diving board.



If h represents the diver's height t seconds after jumping, an equation for the diver's height is $h = 10 + 22t - 32t^2$. In the equation, the 10 gives the height of the diving board, which is where the diver is when $t = 0$. The $22t$ term and the $-32t^2$ term account for the effects of the diver jumping up and gravity pulling the diver down toward the water.

What would the graph look like if the diver made the same jump off a diving board that was 15 feet above the water instead of 10 feet above it?

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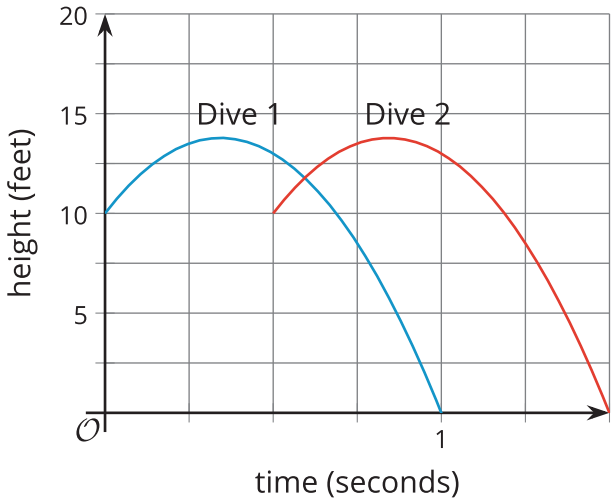


Notice that the graph is moved upward by 5 units. Instead of starting at 10 feet above the water, the diver starts at 15 feet. Instead of a maximum height of close to 14 feet, the maximum height is now close to 19 feet. An equation for the new graph is $h = 15 + 22t - 32t^2$. Notice that only the constant term changed: the 10 increased to 15.

Here is a task to try with your student:

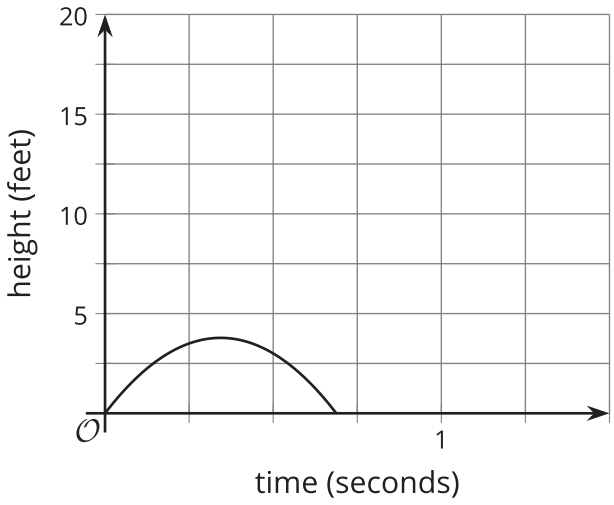
Let's look again at the diver's height represented by the equation $h = 10 + 22t - 32t^2$.

1. If the diver were to make the same jump starting at the level of the water, what equation would give her height?
2. Sketch a graph representing your equation, either by hand or using technology.
3. Use your graph to estimate when the diver would hit the water.
4. When does the diver reach the highest point in the dive? How does this time compare to the time of the high point in the dive when the diver jumps from 10 or 15 feet above the water?
5. Here is the graph of the equation $h = 10 + 22t - 32t^2$, labeled Dive 1, and a second graph for a different dive, labeled Dive 2. How do these two dives compare?



Solution:

1. $h = 22t - 32t^2$.



2.

3. After about $\frac{2}{3}$ of a second

4. Between $\frac{1}{4}$ and $\frac{1}{2}$ second, or about $\frac{1}{3}$ of a second. This is the same time the diver was at the highest point in the other graphs too: the shape of the graph is the same, just shifted vertically.

5. For each of the two dives, the diver starts from 10 feet and reaches a maximum height of close to 14 feet. In the second dive, the diver leaves the diving board a half-second later than the diver in the first dive did.