Unit 5, Lesson 14

Absolute Value Functions (Part 2)

Goals

- Analyze and describe (orally and in writing) the effects of adding a constant term to an expression defining an absolute value function.
- Define absolute value function in terms of the distance of the input from zero.
- Interpret an absolute value function described in words or in function notation, and create a table of values and a graph to represent the function.

Learning Targets

- I can describe the effects of adding a number to the expression that defines an absolute value function.
- I can explain the meaning of absolute value function in terms of distance.
- When given an absolute value function in words or in function notation, I can make sense of it and create a table of values and a graph to represent it.

Lesson Narrative

Students learned in previous courses that **absolute value** is the distance between a number and 0 on the number line. In this lesson, students look at the function $f(x) = |x|$.

Students see that because the graph of the absolute value function is composed of two linear pieces that form a V shape, the same function can also be defined as a piecewise function and described with this equation:

$$f(x) = \begin{cases} x, & x \geq 0 \\ -x, & x < 0 \end{cases}$$

Students also look at basic translations of the absolute value function by considering functions of the forms $f(x) = |x + a|$ and $g(x) = |x| + b$. They focus on the location of the **vertex of the graph**, which is the highest or lowest point in an area of the graph. After noticing the influence of the constants, an optional activity is available to practice translating absolute value functions.

Students look for and make use of structure (MP7) to relate the translation of a graph to the terms or values in the expression that defines the function. The distinctive V shape of the graph of an absolute value function is ideal for observing how adding or subtracting a constant term affects its graph.

Standards

Addressing	**HSF-BF.A.1, HSF-BF.A.1.a, HSF-BF.B.3, HSF-IF.C, HSF-IF.C.7.b**
Building Toward	**HSA-CED.A.1, HSF-BF.A.1**

Instructional Routines

- 5 Practices
- Graph It
- MLR1: Stronger and Clearer Each Time
- MLR8: Discussion Supports

Required Materials

Materials to Gather

- Graphing technology: Activity 4

Required Preparation

Activity 3:

Acquire devices that can run Desmos (recommended) or other graphing technology. (Desmos is available under Math Tools.)

Student Facing Learning Goals

Let's investigate distance as a function.

14.1 Temperature in Toronto

Warm-up

10 min

Activity Narrative

In this *Warm-up*, students begin the transition from thinking about absolute guessing error to thinking about the absolute value function. Using a target value of 0, students see that the function in this activity is equivalent to the distance function.

Standards

Addressing	**HSF-BF.A.1.a, HSF-IF.C.7.b**
Building Toward	**HSA-CED.A.1**

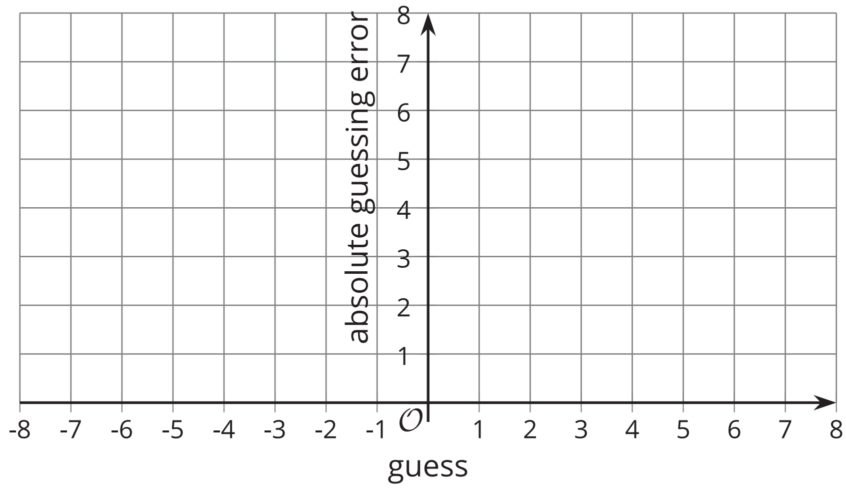
Student Task Statement

Toronto is a city at the border of the United States and Canada, just north of Buffalo, New York. Here are twelve guesses of the average temperature of Toronto, in degrees Celsius, in February 2017.

5 2 -5 3 0 -1 1.5 4 -2.5 6 4 -0.5

1. The actual average temperature of Toronto in February 2017 is 0 degrees Celsius.

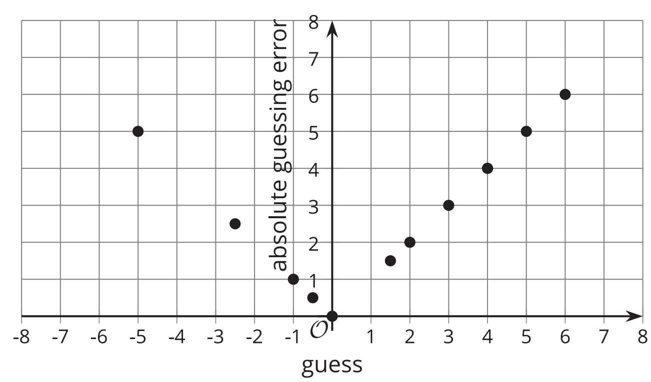
   Use this information to sketch a scatter plot representing the guesses, x, and the corresponding absolute guessing errors, y.



2. What rule can you write to find the output given the input?

Student Response

1. See graph.
2. Sample responses:
    - To find the output, subtract 0 from the input and take the absolute value.
    - To find the output, take the absolute value of the input.



Building on Student Thinking

Some students may struggle to plot the data without explicitly computing the absolute guessing errors first or creating a table of values. Encourage them to take those intermediate steps if they are helpful.

Activity Synthesis

Select a student to display the completed scatter plot. Ask students:

- "How is the scatter plot for this data like the scatter plot for the absolute guessing errors from an earlier lesson?" (The points still form a V shape. There are still no negative y-values.)
- "How are they different?" (The graphs are shifted horizontally toward the vertical axis. The two parts of the *V* now meet at $(0, 0)$.)

To help students see that the "actual average temperature" is like the "actual number of items in a jar" they saw earlier, highlight that:

- Earlier, we saw that when the actual number of items is a, the absolute guessing error is "the distance of guess from a," which can be expressed as "the absolute value of (guess - a)," or $|\text{guess} - a|$.

- Here, likewise, when the actual average temperature is a and the guess is x, the absolute guessing error, y, is "the distance of x from a," which can be written as $y = |x - a|$.
- When the actual temperature in Toronto is 0 degrees Celsius, y is "the distance of x from 0," which can be written as $y = |x - 0|$, or simply $y = |x|$.

14.2 The Distance Function

15 min

Activity Narrative

This activity formally introduces students to the absolute value function as the function that takes an input value and gives its distance from the origin as the output. Students see two different ways to represent this function algebraically: using the absolute value notation and using the cases notation:

$$A(x) = |x|$$

$$A(x) = \begin{cases} x, & x \geq 0 \\ -x, & x < 0 \end{cases}$$

The equation in cases notation focuses on what has to happen algebraically to an input value to give its distance from the origin. Different rules apply to different intervals of the domain.

As students work, look for those who plot points for the graph of function A and those who sketch two lines. Ask students who sketch different types of graphs to share during discussion.

Standards

Addressing	**HSF-IF.C, HSF-IF.C.7.b**
Building Toward	**HSA-CED.A.1, HSF-BF.A.1**

Instructional Routines

- MLR1: Stronger and Clearer Each Time

Launch

Arrange students in groups of 2. Give students a few minutes of quiet work time, than time to share their response with their partner. Follow with a whole-class discussion.

Access for English Language Learners

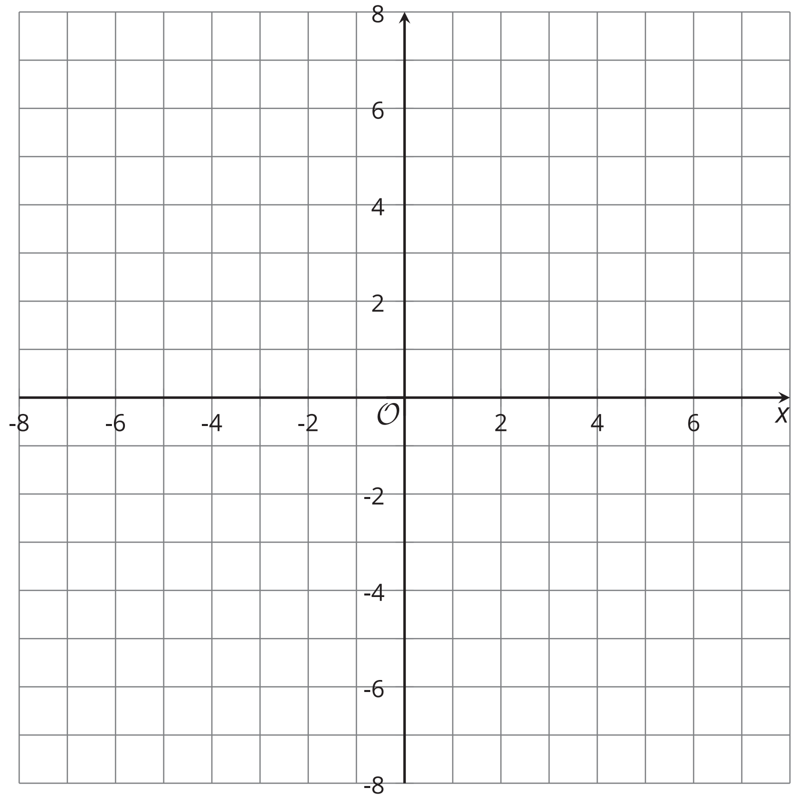
MLR1 Stronger and Clearer Each Time. Before the whole-class discussion, give students time to meet with 2–3 partners to share and get feedback on their first draft response to the last question. Invite listeners to ask questions and give feedback that will help their partner clarify and strengthen their ideas and writing. Give students 3–5 minutes to revise their first draft based on the feedback they receive.
Advances: Writing, Speaking, Listening

Student Task Statement

The function A gives the distance of x from 0 on the number line.

1. Complete the table with at least one possible value in each blank position, and sketch a graph of function A.

x	$A(x)$
8	
	5.6
π	
$\frac{1}{2}$	
	1
0	
$-\frac{1}{2}$	
-1	
-5.6	
	8



2. Andre and Elena are trying to write a rule for this function.

   - Andre writes: $A(x) = \begin{cases} x, & x \geq 0 \\ -x, & x < 0 \end{cases}$
   - Elena writes: $A(x) = |x|$

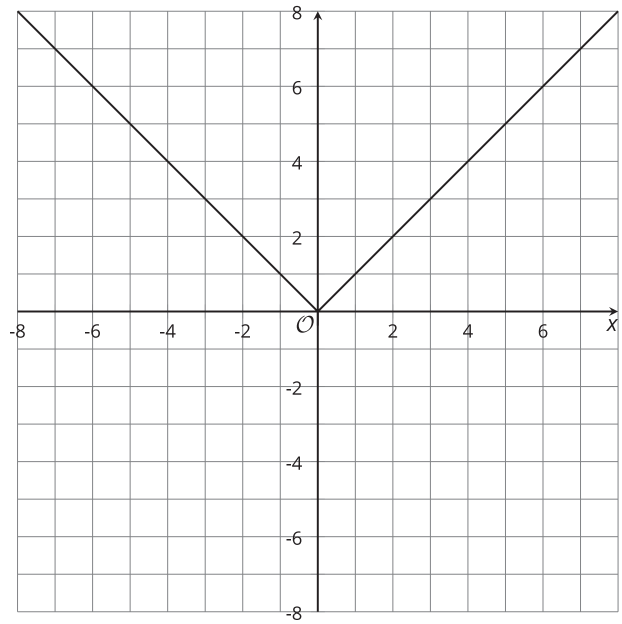
   Explain why both equations correctly represent the function A.

Student Response

1.

x	$A(x)$
8	8
-5.6 or 5.6	5.6
π	π
$\frac{1}{2}$	$\frac{1}{2}$
-1 or 1	1

x	$A(x)$
0	0
$-\frac{1}{2}$	$\frac{1}{2}$
-1	1
-5.6	5.6
-8 or 8	8



2. Sample response:
    - Andre's rule says that when x is positive or 0, its distance from 0 is just x, which is positive. This makes sense because distance is always positive. When x is negative, we cannot write x for its distance from 0 because it would be a negative value. To find the distance, we need the opposite of that negative value, or $-x$.
    - Elena's rule says that distance of x from 0 is the absolute value of x, which is always positive.

Building on Student Thinking

Students who interpret the "$-x$" in Andre's rule to mean "negative x" (rather than "the opposite of x") may be unsure how to use that information. Ask students to evaluate the function for specific values of x and to write down each step. For instance, when x is -2, $A(-2)$ is $-(-2)$, which is 2.

Activity Synthesis

Select previously identified students to display their graph of function A. Ask students who plotted only the ordered pairs in the table whether the pairs that are not in the table, if plotted, would also fall on the same two lines. Emphasize that A can be shown with two lines.

Next, ask students to share their response to the last question. Students may find Elena's rule easier to explain because of their work with absolute value in recent activities. They may struggle to explain Andre's rule.

We can reason about Andre's equation a couple of ways:

- By thinking about rules: The equation is that of a piecewise function because different rules are applied to different parts of the domain to give "the distance from 0" as the output:
    - When the input x is positive or 0, its distance from 0 is that same number, x.
    - When the input x is negative, its distance from 0 is the opposite of the number, $-x$.
- By using the graph: The two halves of the graph are lines with different slopes.
    - If we cover up the left side of the vertical axis and see only positive values of x, we see a line with a slope of 1

that represents $y = x$.

- If we cover up the right side and see only negative values of x, we see a line with a slope of -1 that represents $y = -x$

Explain to students that:

- Function A is the **absolute value** function. It gives the distance of an input value from a certain value, the origin in this case.
- The graph of function A is a V shape with the two lines of points meeting at $(0, 0)$, which is the minimum of the graph. We call this point the **vertex** of the graph. It is the point where the graph changes direction.
- The absolute value function is also a piecewise function because different rules are applied to different parts of the domain to get the outputs.

Access for Students with Disabilities

Representation: Develop Language and Symbols. Create a display of important terms and vocabulary. Invite students to suggest language or diagrams to include that will support their understanding of the absolute value function. Terms may include “absolute value function,” “vertex,” “cases notation.”
Supports accessibility for: Conceptual Processing, Language

10 min

14.3 Moving Graphs Around

Activity Narrative

There is a digital version of this activity.

In this activity, students expand their awareness of absolute value functions by analyzing the graphs and equations of several absolute value functions. The graphs have the same V shape but not the same vertex. The expressions that define the functions have a constant term added to or subtracted from the input, x, or from the absolute value of the input, $|x|$. Students observe how each constant term affects the graph and consider possible explanations.

Dynamic graphing technology can be very useful for observing how the parameters of a function are related to the features of its graph and can help students generalize their observations. Consider giving students individual access to dynamic graphing technology and then giving them time to explain the behaviors of the graphs. If students don't have individual access, projecting the applet in the digital *Launch* would be helpful.

Although digital tools can help students notice the connection between the parameters and the graph easily, it is still important for students to understand why they behave in that way.

Monitor for students who use these different strategies:

- Find some input-output pairs for each function and verify that the graph contains those pairs of values.
- Interpret the expressions in terms of finding an absolute error of a guess.
- Use the position of the number added or subtracted to reason about how the inputs and outputs are affected in the graph.

Plan to have students present in this order to support moving students from plotting points to thinking of functions as an object that can be translated.

In the digital version of the activity, students use an applet to make dynamic graphs and see how adjusting parameters translates functions. The applet allows students to see how the different values affect the graph in real time. Use the digital version if available so that students can more easily see the connections between the values added to different parts of the function and the result for the graph.

Standards

Addressing **HSF-BF.B.3, HSF-IF.C.7.b**

Instructional Routines

- 5 Practices
- Graph It

Launch

Give students a few minutes of quiet think time. Provide access to graphing technology, if requested.

Ask students to not only observe how the addition or subtraction of 2 affects each graph, but also be prepared to offer an explanation for why it makes sense that the graph is where it is. Consider demonstrating what a possible explanation could look or sound like, using function f as an example. Or, consider giving prompts, such as:

- It makes sense that the graph of ________ is located ________ because . . . .
- I know that the vertex of the graph of ________ belongs at ________ because . . . .

Select students with different approaches, such as those described in the *Activity Narrative*, to share later.

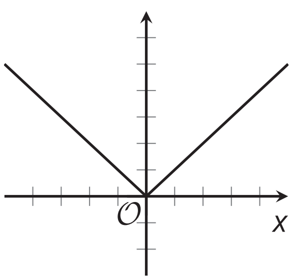
Access for Students with Disabilities

Action and Expression: Provide Access for Physical Action. Provide access to tools and assistive technologies, such as a graphing calculator or graphing software. Some students may benefit from a checklist or list of steps to be able to use the calculator or software.
Supports accessibility for: Organization; Conceptual processing; Attention

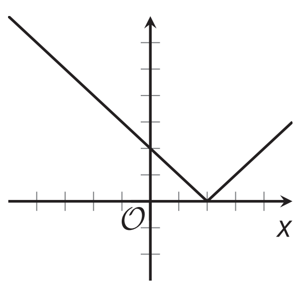
Student Task Statement

Here are equations and graphs that represent five **absolute value** functions.

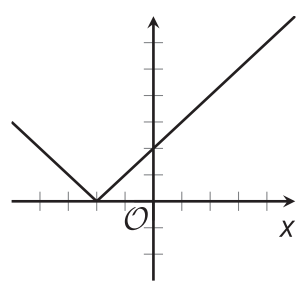
$f(x) = |x|$



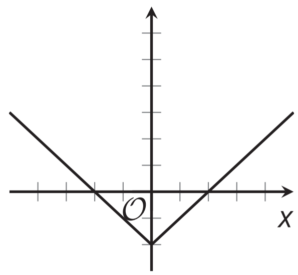
$g(x) = |x - 2|$



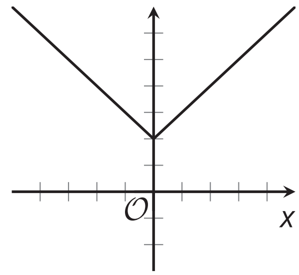
$h(x) = |x + 2|$



$j(x) = |x| - 2$



$k(x) = |x| + 2$



Notice that the number 2 appears in the equations for functions g, h, j, and k. Describe how the addition or subtraction of 2 affects the graph of each function.

Then, think about a possible explanation for the position of the graph. How can you show that it really belongs where it is on the coordinate plane?

Student Response

See *Activity Narrative* for possible explanations.

- Function g: Subtracting 2 from x moves the *V*-shaped graph to the right by 2 units.
- Function h: Adding 2 to x moves the graph to the left by 2 units.
- Function j: Subtracting 2 from $|x|$ moves the *V*-shaped graph down by 2 units.
- Function k: Adding 2 to $|x|$ moves the graph up by 2 units.

Are You Ready for More?

1. Mark the minimum of each graph in the activity. Each point you marked represents the least output value of the function.

   In each function, what value of x gives that minimum output value?

2. a. Another function is defined by $m(x) = |x + 11.5|$. What value of x produces the least output of function m? Be prepared to explain how you know.

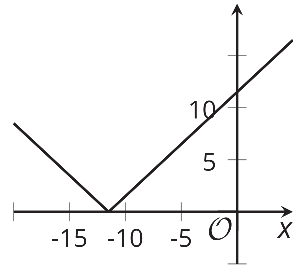
   b. Describe or sketch the graph of m.

Extension Student Response

1. The vertex of each graph should be marked. The value of x that gives the least output in each function:
   - $x = 0$ in f
   - $x = 2$ in g
   - $x = -2$ in h
   - $x = 0$ in j and k

2. a. $x = -11.5$ gives the least value of $m(x)$. The absolute value of a number cannot be negative, so the least possible value of $|x + 11.5|$ must be 0.

   b. Sample responses: It's the graph of f shifted 11.5 units to the left.



Activity Synthesis

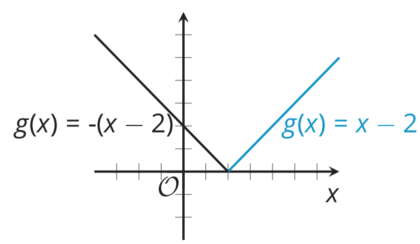
Invite previously selected students to share their reasoning about the position of the graphs with different parameters. Sequence the discussion of the approaches by the order listed in the *Activity Narrative*. If possible, record and display their work for all to see.

Connect the different responses to the learning goals by asking questions, such as:

- "In a guessing game, if the target number is 2, would the absolute guessing error be calculated using $|x - 2|, |x + 2|, |x| + 2,$ or $|x| - 2$?" (It would be $|x - 2|$.)
- "If the absolute guessing error function for the target number of 2 is graphed, where is the vertex? Use your strategy from this activity to explain." (The vertex is at $(2, 0)$. This is because the graph will look like $y = |x|$, but shifted to the right 2.)
- "How does the function and graph change if the target number is -2?" (It would be $|x + 2|$, and the graph would have a similar shape, but with the vertex at $(-2, 0)$.)

If time permits and if not already mentioned by students, remind students that each function can be seen as a piecewise function with two parts, each part being a linear function.

Graphing the two linear functions gives two lines that intersect on the horizontal axis. For g, the two lines meet at $(2, 0)$. For h, they meet at $(-2, 0)$.



14.4 More Moving Graphs Around

Optional

15 min

Activity Narrative

This optional activity gives students a chance to test the observations or apply the generalizations they made in the previous activity. They match equations and graphs of other absolute value functions, then sketch the graph of an equation without a match.

Students also encounter new cases in which a constant term is added or subtracted both before and after absolute value is applied to the input, resulting in a graph that shifts both vertically and horizontally relative to the graph of $f(x) = |x|$.

In making the matches (and before using technology to check their graphs), students are likely to reason in different ways and rely on structure to varying degrees (MP7). For instance, students may:

- Evaluate each function at different input values and then match the input-output pairs to the coordinates on the graphs.

- Start with the graph of $|x|$ and then shift sideways for addition or subtraction inside the absolute value symbols, and then shift the graph vertically for addition or subtraction after absolute value is applied.
- Think about the x-value that would produce the smallest possible output and relate that ordered pair to the vertex of the graph.

Monitor for students using different strategies, and invite them to share during discussion.

Students will have opportunities to explore transformations of functions and their graphs in a later unit and more formally in a future course.

Standards

Addressing **HSF-BF.B.3, HSF-IF.C.7.b**

Instructional Routines

- Graph It
- MLR8: Discussion Supports

Launch

Provide access to devices that can run Desmos or other graphing technology.

Access for English Language Learners

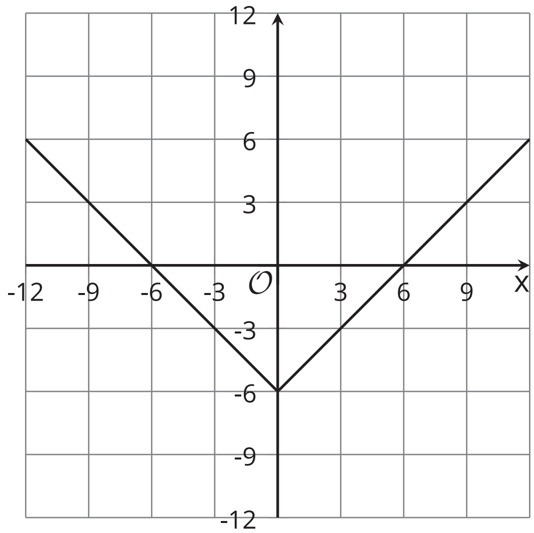
MLR8 Discussion Supports. Students should take turns finding a match and explaining their reasoning to their partner. Display the following sentence frames for all to see: "I noticed ________, so I matched ________." Encourage students to challenge each other when they disagree.
Advances: Speaking, Listening

Student Task Statement

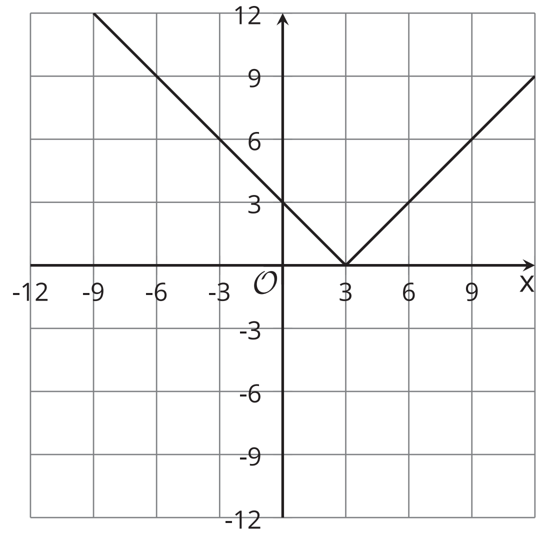
Here are five equations and four graphs.

- Equation 1: $y = |x - 3|$
- Equation 2: $y = |x - 9| + 3$
- Equation 3: $y = |x| - 6$
- Equation 4: $y = |x + 3|$
- Equation 5: $y = |x + 3| - 6$

A



B



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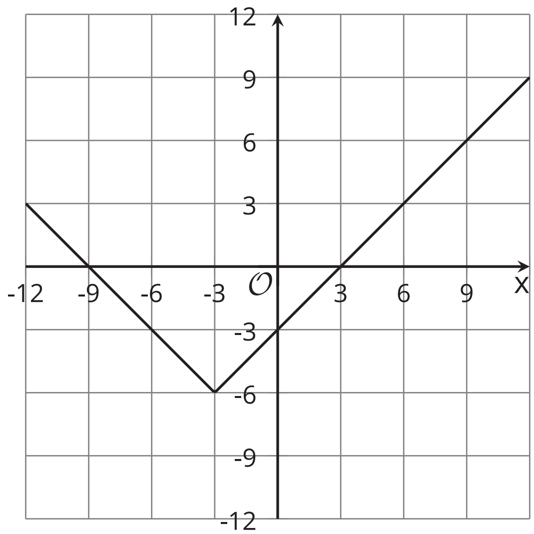
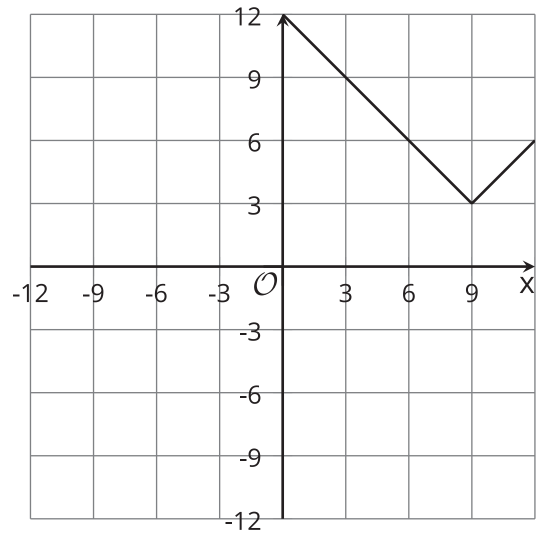
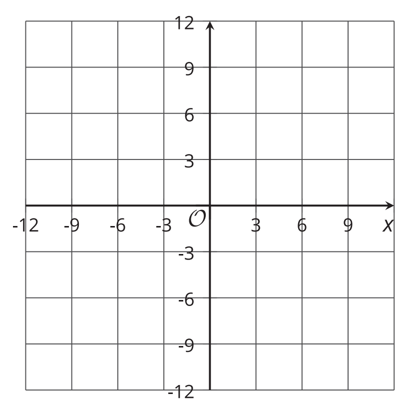


D



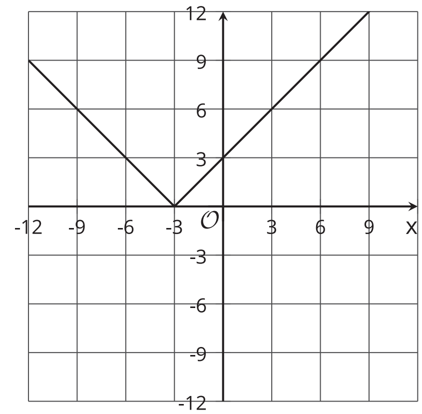
1. Match each equation with a graph that represents it. One equation has no match.
2. For the equation without a match, sketch a graph on the blank coordinate plane.
3. Use graphing technology to check your matches and your graph. Revise your matches and graphs as needed.



Student Response

1. a. Equation 1: Graph B
   b. Equation 2: Graph D
   c. Equation 3: Graph A
   d. Equation 4: no match
   e. Equation 5: Graph C
2. See graph.
3. No response required.

Graph of $y = |x + 3|$



Activity Synthesis

Select previously identified students to share their strategies for making a match. If students use the strategies listed in the *Activity Narrative*, order their presentation as shown.

If time permits, ask students to use a strategy that they find effective to describe the graphs of $f(x) = |x - 3| + 5$ and $y = |x + 4| + 7$.

Lesson Synthesis

Display the graph of $A(x) = |x|$ for all to see, along with the two equations for the function, $A(x) = |x|$ and
$$A(x) = \begin{cases} x, & x \geq 0 \\ -x, & x < 0 \end{cases}$$

Ask questions such as:

- "None of the points on the graph of this absolute value function lie below the x-axis. Why is that?" (The absolute value of a number is never negative.)
- "Suppose we know that $A(x)$ is 4. How do we know what value or values of x would give an output of 4?" (We look at numbers that are 4 units from 0. There are two numbers that meet this requirement: 4 and -4.)
- "How do we use the equation $A(x) = |x|$ to find the function value at $x = -5$?" ($A(-5) = |-5|$, which is 5.)
- "How do we use the equation that uses the cases notation to find the function value at $x = -5$?" (Because -5 is less than 0, we use the rule for $x < 0$ and find $A(-5)$, which gives $-(-5)$ or 5.)

14.5 Elevations of Places

Cool-down

5 min

Standards

Addressing **HSF-BF.A.1, HSF-IF.C**

Launch

Before students begin the *Cool-down*, consider reading the opening paragraph as a class. Clarify the meaning of "elevation" and "sea level" as needed.

Student Task Statement

The term "elevation" is often used to describe the height of a place (such as a city, a mountain, or a valley) compared to sea level. For example, the highest point of Houston, Texas has an elevation of 105 feet. The surface of the sea has an elevation of 0 feet. Some places are below sea level, so their elevations are negative values.

1. The table shows the elevation, e, of several towns.

   Function f gives the vertical distance of each town from sea level. Both e and $f(e)$ are measured in feet. Complete the table of values.

e	180	12.1	5.4		-5.4	-36	-180
$f(e)$				0			

2. Write an equation to represent $f(e)$.
3. Two towns have different elevations, but when the elevations are used as inputs of $f(e)$, they both produce an output of 25.

   What are the elevations of the two towns? Why do they produce the same output?

Student Response

1.

e	180	12.1	5.4	0	-5.4	-36	-180
$f(e)$	180	12.1	5.4	0	5.4	36	180

2. $f(e) = |e|$
3. The elevations are 25 feet and -25 feet. The two towns are both the same distance from sea level, but in opposite directions (one town is above sea level, the other below).

Responding to Student Thinking

Press Pause
If most students struggle with making sense of the absolute value function, make time to revisit it. For example, review related work in the practice problems of the lesson referred to here. See the Course Guide for ideas to help students re-engage with earlier work.

Algebra 1, Unit 5, Lesson 14 Absolute Value Functions (Part 2)

Lesson 14 Summary

In a guessing game, each guess can be seen as an input of a function and each absolute guessing error as an output. Because absolute guessing error tells us how far a guess is from a target number, the output is distance.

Suppose the target number is 0.

- We can find the distance of a guess, x, from 0 by calculating $x - 0$. Because distance cannot be negative, what we want to find is $|x - 0|$, or simply $|x|$.
- If function f gives the distance of x from 0, we can define it with this equation:

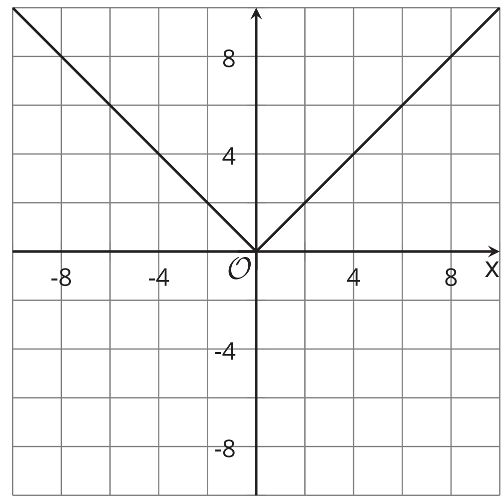
$$f(x) = |x|$$

Function f is the **absolute value** function. It gives the distance of x from 0 by finding the absolute value of x.

The graph of function f is a V shape with the two lines converging at $(0, 0)$.

We call this point the **vertex** of the graph. It is the point where a graph changes direction, from going down to going up, or the other way around.

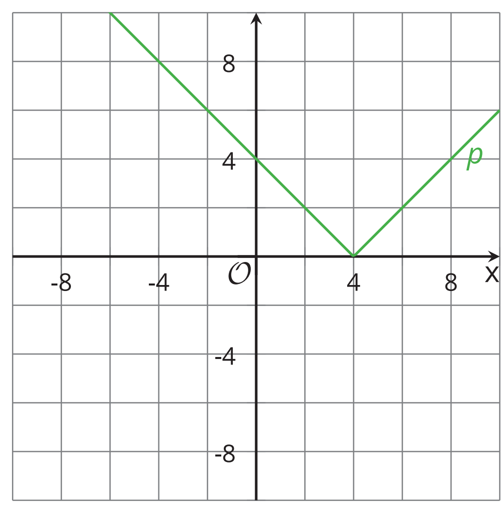
We can also think of a function like f as a *piecewise function* because different rules apply when x is less than 0 and when x is greater than 0.



Suppose we want to find the distance between x and 4.

- We can find the difference between x and 4 by calculating $x - 4$. Distance cannot be negative, so what we want is the absolute value of that difference: $|x - 4|$.
- If function p gives the distance of x from 4, we can define it with this equation:

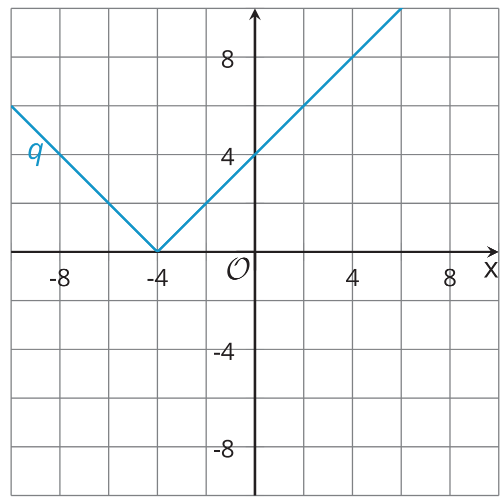
$$p(x) = |x - 4|$$



Now suppose we want to find the distance between x and -4.

- We can find the difference of x and -4 by calculating $x - (-4)$, which is equal to $x + 4$. Distance cannot be negative, so let's find the absolute value: $|x + 4|$.
- If function q gives the distance of x from -4, we can define it with this equation:

$$q(x) = |x + 4|$$



Notice that the graphs of p and q are like that of f, but they have shifted horizontally.

Glossary

- absolute value
- vertex (of a graph)

Lesson 14 Practice Problems

1 Student Task Statement

The absolute value function can be defined using piecewise notation.

$$A(x) = \begin{cases} x, & x \geq 0 \\ -x, & x < 0 \end{cases}$$

Use this notation to find the values:

a. $A(10)$
b. $A(0)$
c. $A(-3)$
d. $A(3.14159)$

Solution

a. 10
b. 0
c. 3
d. 3.14159

2 Student Task Statement

Here are four equations of absolute value functions and three coordinate pairs. Each coordinate pair represents the vertex of the graph of an absolute value function.

Match the equation of each function with the coordinates of the vertex of its graph. The vertex coordinates of the graph of one equation are not shown.

A. $p(x) = |x - 9|$
B. $q(x) = |x| + 9$
C. $r(x) = |x + 9|$
D. $t(x) = |x| - 9$

1. $(-9, 0)$
2. $(9, 0)$
3. $(0, -9)$

Solution

- A matches 2
- B matches No match
- C matches 1
- D matches 3

3

Student Task Statement

Function G is defined by the equation $G(x) = |x|$.

Function R is defined by the equation $R(x) = |x| + 2$.

Describe how the graph of function R relates to the graph of G, or sketch the graphs of the two functions to show their relationship.

Solution

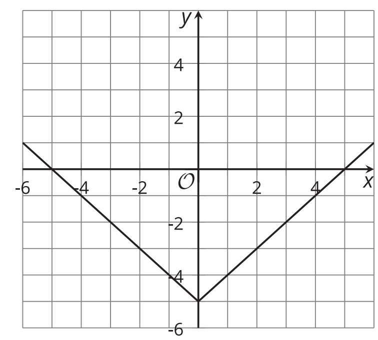
Sample response: Graph of G is a V shape, with its vertex at the origin. If we shift it up 2 units, we have the graph of function R.

4

Student Task Statement

Here is the graph of a function.

Select the equation for the function represented by the graph.



A. $y = |x| - 5$

B. $y = |x| + 5$

C. $y = |x - 5|$

D. $y = |x + 5|$

Solution

A

5

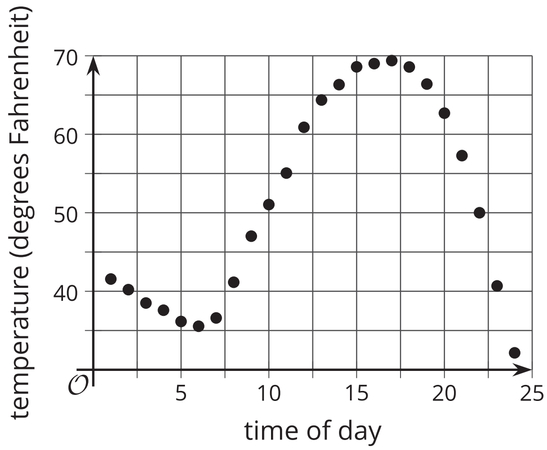
from Unit 5, Lesson 7

Student Task Statement

The temperature was recorded at several times during the day. Function T gives the temperature in degrees Fahrenheit, n hours since midnight. Here is a graph for this function.

a. Pick two consecutive points, and connect them with a line segment.

Estimate the slope of that line. Explain what that estimated value means in this situation.



b. Pick two nonconsecutive points, and connect them with a line segment. Estimate the slope of that line. Explain what that estimated value means in this situation.

Solution

a. Sample response: A line connecting $(8, 41)$ and $(9, 47)$ is drawn. The slope is $\frac{47-41}{9-8} = 6$. The slope tells us that between 8 a.m. and 9 a.m., the average rate of change in temperature is 6 degrees Fahrenheit per hour.

b. Sample response: A line connecting $(5, 36)$ and $(15, 69)$ is drawn. The slope is $\frac{69-36}{15-5} = 3.3$. The slope tells us that between 5 a.m. and 3 p.m., the average rate of change in temperature is 3.3 degrees Fahrenheit per hour.

6 from Unit 5, Lesson 8

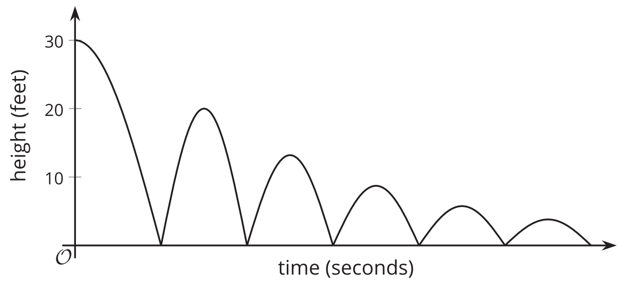
Student Task Statement

A tennis ball is dropped from an initial height of 30 feet. It bounces 5 times, with each bounce height being about $\frac{2}{3}$ of the height of the previous bounce.

Sketch a graph that models the height of the ball over time. Be sure to label the axes.

Solution

Sample response:

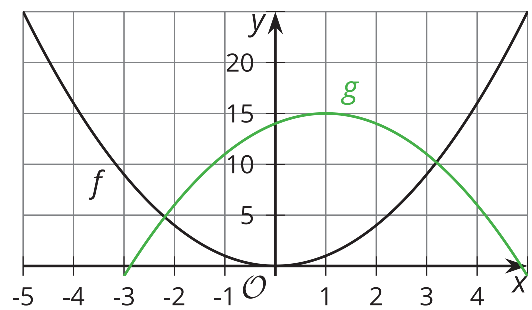


7 from Unit 5, Lesson 9

Student Task Statement

Here are two graphs representing functions f and g.

Identify at least two values of x at which the inequality $g(x) > f(x)$ is true.



Solution

Sample response: $x = -1$ and $x = 2$