



# Representing Exponential Growth

Let's explore exponential growth.

## 3.1 Math Talk: Exponent Rules

Rewrite each expression as a power of 2.

•  $2^3 \cdot 2^4$

•  $2^5 \cdot 2$

•  $2^{10} \div 2^7$

•  $2^9 \div 2$

## 3.2 What Does $x^0$ Mean?

1. Complete the table. Take advantage of any patterns you notice.

|       |    |    |   |   |   |
|-------|----|----|---|---|---|
| $x$   | 4  | 3  | 2 | 1 | 0 |
| $3^x$ | 81 | 27 |   |   |   |

2. Here are some equations. Find the solution to each equation using what you know about exponent rules. Be prepared to explain your reasoning.

a.  $9^? \cdot 9^7 = 9^7$

b.  $\frac{9^{12}}{9^?} = 9^{12}$

3. What is the value of  $5^0$ ? What about  $2^0$ ?

### Are you ready for more?

We know, for example, that  $(2 + 3) + 5 = 2 + (3 + 5)$  and  $2 \cdot (3 \cdot 5) = (2 \cdot 3) \cdot 5$ . The grouping with parentheses does not affect the value of the expression.

Is this true for exponents? That is, are the numbers  $2^{(3^5)}$  and  $(2^3)^5$  equal? If not, which is bigger? Which of the two would you choose as the meaning of the expression  $2^{3^5}$  written without parentheses?

### 3.3 Multiplying Microbes

1. In a biology lab, 500 bacteria reproduce by splitting. Every hour, on the hour, each bacterium splits into two bacteria.

- a. Write an expression to show how to find the number of bacteria after each hour listed in the table.

| hour | number of bacteria |
|------|--------------------|
| 0    | 500                |
| 1    |                    |
| 2    |                    |
| 3    |                    |
| 6    |                    |
| $t$  |                    |

- b. Write an equation relating  $n$ , the number of bacteria, to  $t$ , the number of hours.

- c. Use your equation to find  $n$  when  $t$  is 0. What does this value of  $n$  mean in this situation?

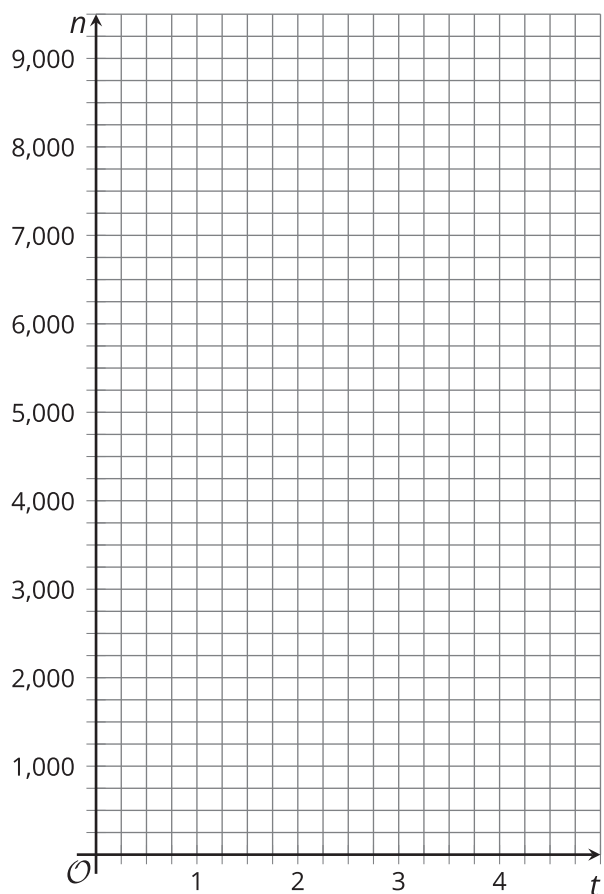
- d. When the values of one variable are multiplied by the same value each time the other variable increases by 1, that multiplier is called the growth factor. What is the **growth factor** in this situation?

2. In a different biology lab, a population of single-cell parasites also reproduces hourly. An equation that gives the number of parasites,  $p$ , after  $t$  hours is  $p = 100 \cdot 3^t$ . Explain what the numbers 100 and 3 mean in this situation.

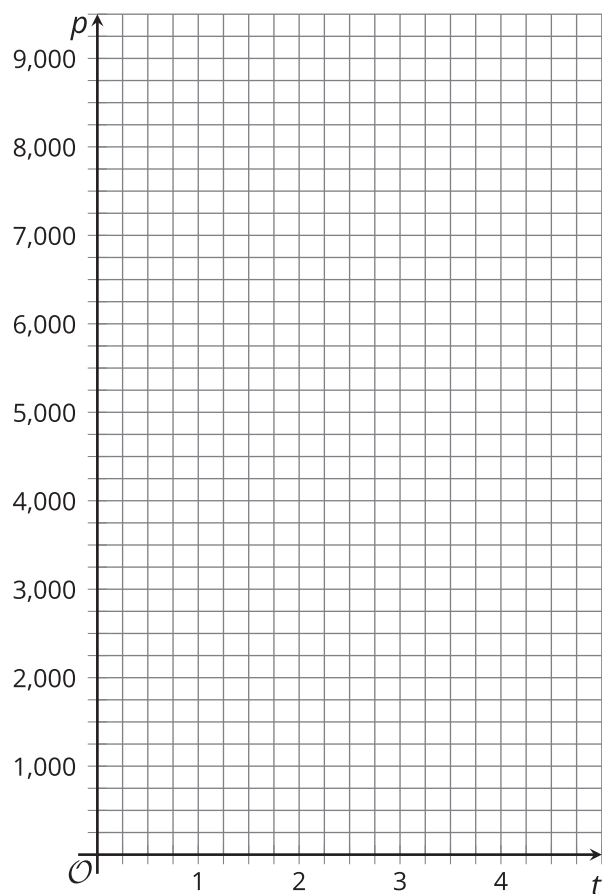
## 3.4 Graphing the Microbes

1. Refer back to your work in the table of the previous task. Use that information and the given coordinate planes to graph the following:

a. Graph  $(t, n)$  when  $t$  is 0, 1, 2, 3, and 4.



b. Graph  $(t, p)$  when  $t$  is 0, 1, 2, 3, and 4.  
(If you get stuck, you can create a table.)



2. On the graph of  $n$ , where can you see each number that appears in the equation?
3. On the graph of  $p$ , where can you see each number that appears in the equation?

### Lesson 3 Summary

In relationships where the change is exponential, a quantity is repeatedly multiplied by the same amount. The multiplier is called the **growth factor**.

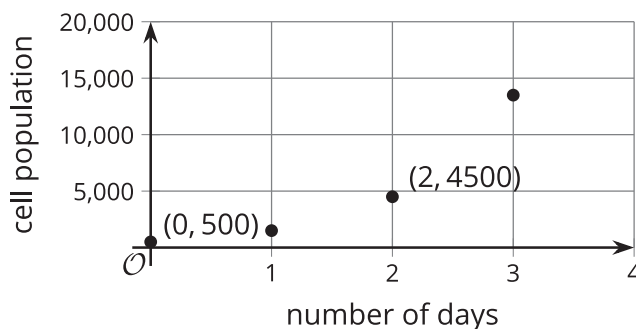
Suppose a population of cells starts at 500 and triples every day. The number of cells each day can be calculated as follows:

| number of days | number of cells                                                 |
|----------------|-----------------------------------------------------------------|
| 0              | 500                                                             |
| 1              | 1,500 (or $500 \cdot 3$ )                                       |
| 2              | 4,500 (or $500 \cdot 3 \cdot 3$ , or $500 \cdot 3^2$ )          |
| 3              | 13,500 (or $500 \cdot 3 \cdot 3 \cdot 3$ , or $500 \cdot 3^3$ ) |
| $d$            | $500 \cdot 3^d$                                                 |

We can see that the number of cells ( $p$ ) is changing exponentially, and that  $p$  can be found by multiplying 500 by 3 as many times as the number of days ( $d$ ) since the 500 cells were observed. The *growth factor* is 3. To model this situation, we can write this equation:  $p = 500 \cdot 3^d$ .

The equation can be used to find the population on any day, including day 0, when the population was first measured. On day 0, the population is  $500 \cdot 3^0$ . Since  $3^0 = 1$ , this is  $500 \cdot 1$  or 500.

Here is a graph of the daily cell population. The point (0, 500) on the graph means that on day 0, the population starts at 500.



Each point is 3 times higher on the graph than the previous point. (1, 1500) is 3 times higher than (0, 500), and (2, 4500) is 3 times higher than (1, 1500).