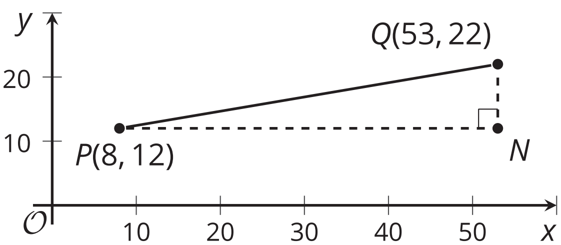
Unit 5, Lesson 8

Distances and Triangles

Let's build an equation for distance.

8.1 Going the Distance

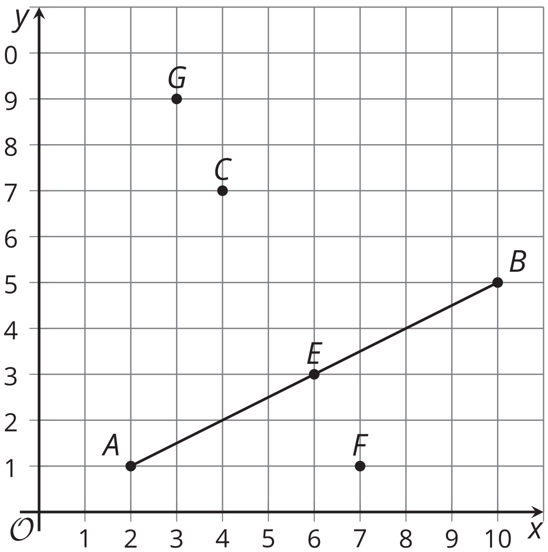


Andre says, “I know that I can find the distance between two points in the plane by drawing in a right triangle and using the Pythagorean Theorem. But I’m not sure how to find the lengths of the legs of the triangle when I can’t just count the squares on the graph.”

Explain to Andre how he can find the lengths of the legs in the triangle in the image. Then, calculate the distance between points P and Q.

8.2 Plenty of Points

The figure shows segment AB and several points.



1. Calculate the exact distance from each point to the endpoints of segment AB.

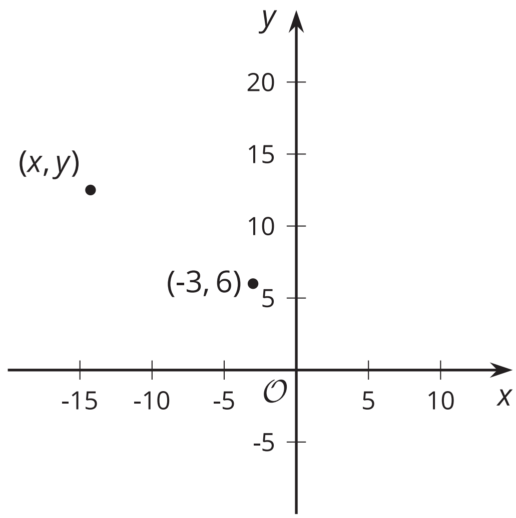
	point A	point B
point G		
point C		
point E		
point F		

2. Calculate the slopes of line segments AB and FG.

3. What do you notice about the distances of the points and the slopes of line segments? What does this tell you about the points G, C, E, and F?

8.3 Building an Equation for Distance

Here is a point at $(-3, 6)$ and a point (x, y).

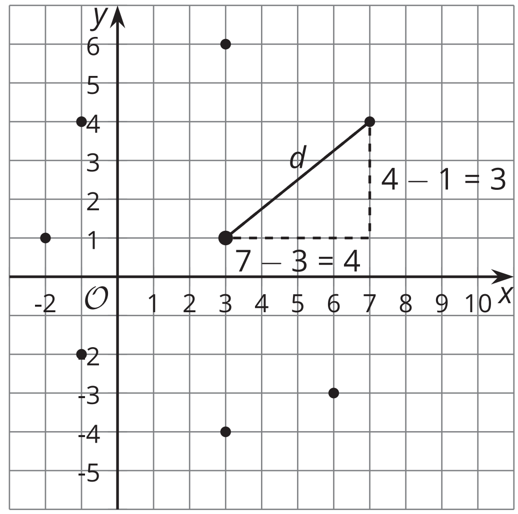


1. Write an equation that would allow you to determine whether a particular point (x, y) is located a distance of 13 units from $(-3, 6)$.

2. Use your equation to test whether $(9, 1)$ is also located a distance of 13 units from $(-3, 6)$.

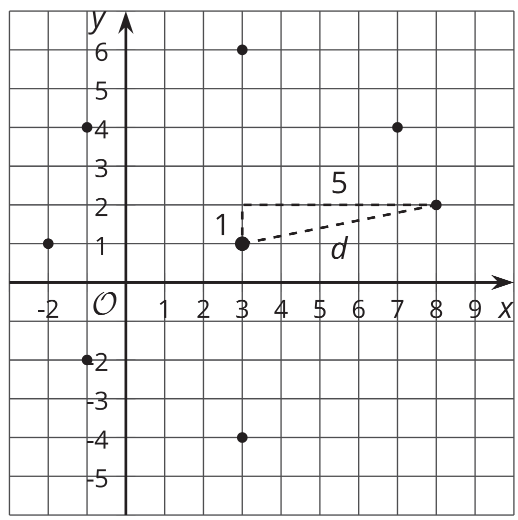
3. Suppose you have a point at (h, k) and a distance d. Write an equation that would allow you to test whether a particular point (x, y) is located d units from (h, k).

Lesson 8 Summary



The diagram shows the point $(3, 1)$, along with several points that are 5 units away from $(3, 1)$. For some points, it is easy to see their distance from $(3, 1)$. For example, we can count or subtract to see that $(-2, 1)$, $(3, 6)$, and $(3, -4)$ are all 5 units away from $(3, 1)$.

For other points, we need to calculate the distance. Let's look at the point $(7, 4)$. To calculate the distance from $(7, 4)$ to $(3, 1)$, we can let d stand for the distance, and set up the Pythagorean Theorem: $(7 - 3)^2 + (4 - 1)^2 = d^2$. Evaluate the left-hand side to find that $25 = d^2$. Now d is the positive number that squares to make 25, which means that $(7, 4)$ really is 5 units away from $(3, 1)$.



The point $(8, 2)$ also looks like it could be 5 units away from $(3, 1)$. To find its distance from $(3, 1)$, we can do a similar calculation: $(8 - 3)^2 + (2 - 1)^2 = d^2$. Evaluating the left side, we get $26 = d^2$. This means that d must be a little more than 5. So $(8, 2)$ is not exactly 5 units away from $(3, 1)$.

To check if any point (x, y) is 5 units away from the point $(3, 1)$, we can use the Pythagorean Theorem to see if $(x - 3)^2 + (y - 1)^2$ is equal to 5^2 or 25.

By the same reasoning, we can check to see if the distance between any points, (x, y) and (h, k), is equal to a distance, d, using the equation $(x - h)^2 + (y - k)^2 = d^2$. When we solve this equation for d, we sometimes call this the *distance formula*: $d = \sqrt{(x - h)^2 + (y - k)^2}$