Reasoning about Solving Equations (Part 1)

Let's see how a balanced hanger is like an equation and how moving its weights is like solving the equation.

6.1 Hanger Diagrams

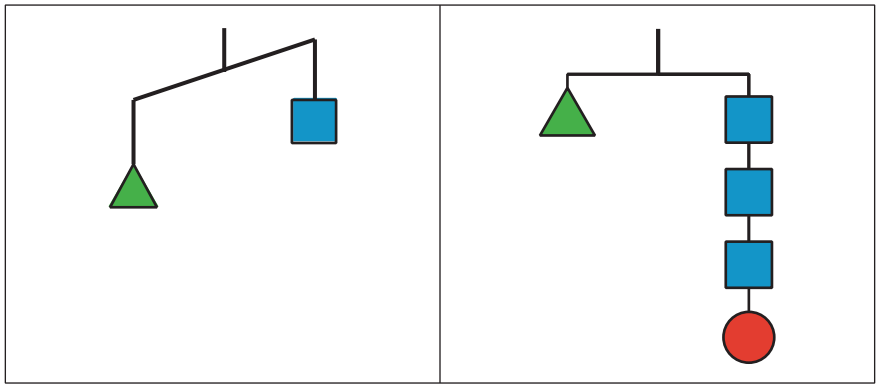
In the two diagrams, all the triangles weigh the same and all the squares weigh the same.

For each diagram, come up with . . .

1. One thing that *must* be true

2. One thing that *could* be true

3. One thing that *cannot possibly* be true



6.2 Hanger and Equation Matching

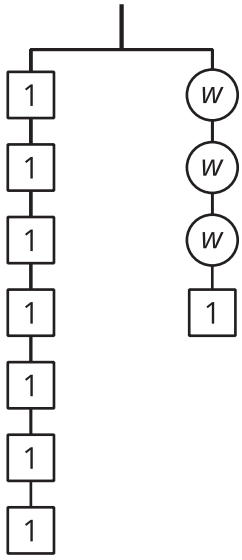
On each balanced hanger, shapes with the same variable have the same weight.

1. Match each hanger to an equation. Complete the equation by writing x, y, z, or w in the empty box.

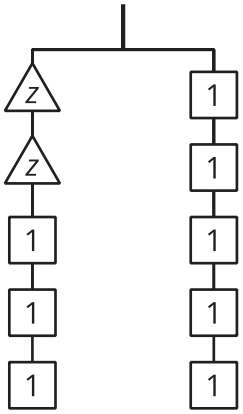
   - $2\square + 3 = 5$
   - $3\square + 2 = 3$
   - $6 = 2\square + 3$
   - $7 = 3\square + 1$

2. Find the solution to each equation. Use the hanger to explain what the solution means.

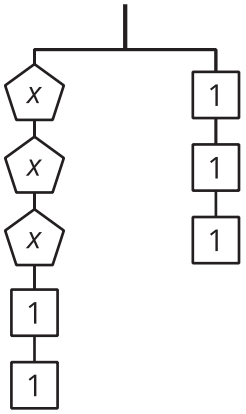
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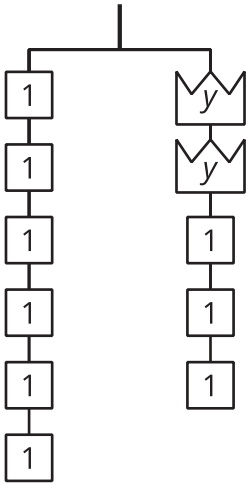
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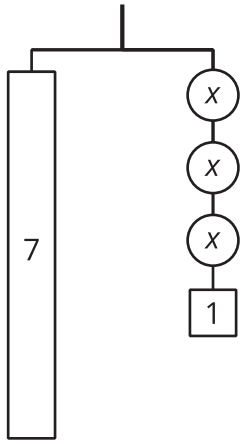


6.3 Use Hangers to Understand Equation Solving

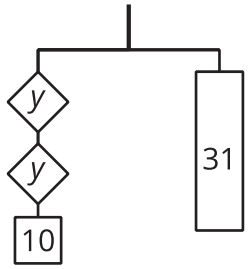
Here are some balanced hangers diagrams where each piece is labeled with its weight in the same units. For each diagram:

1. Write an equation.
2. Explain how to figure out the weight of a piece labeled with a variable by reasoning about the diagram.
3. Explain how to figure out the weight of a piece labeled with a variable by reasoning about the equation.

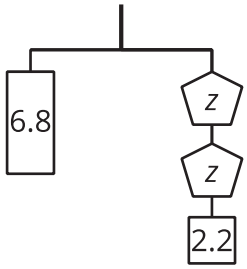
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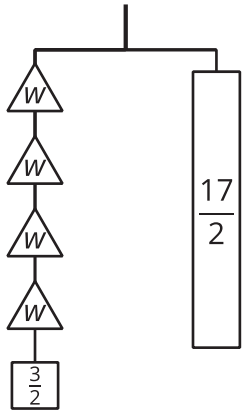
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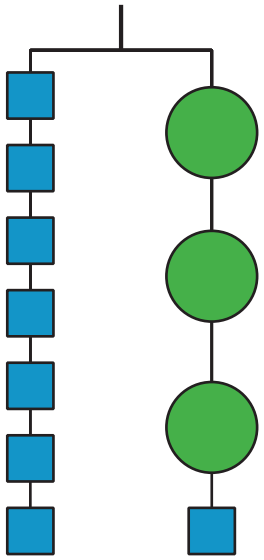
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Lesson 6 Summary

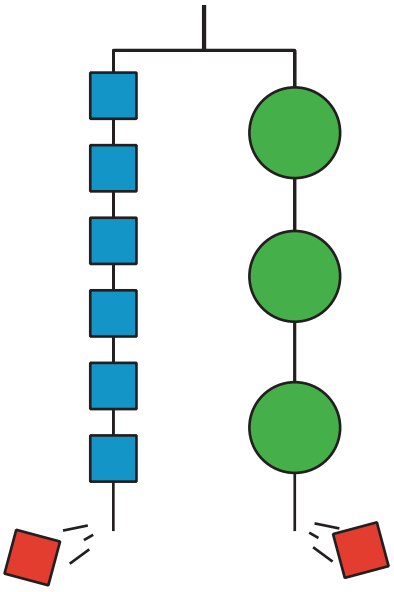
In this lesson, we worked with two ways to show that two amounts are equal: a balanced hanger and an equation. We can use think about the weights on a balanced hanger to understand steps we can use to find an unknown amount in a matching equation.

This hanger diagram shows a total weight of 7 units on one side that is balanced with 3 equal, unknown weights and a 1-unit weight on the other. An equation that represents the relationship is $7 = 3x + 1$.



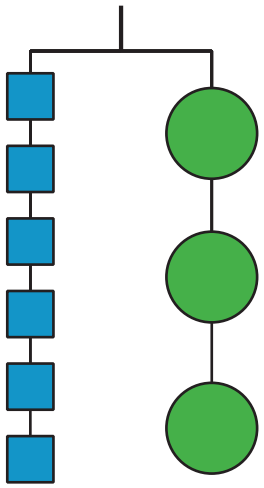
$7 = 3x + 1$

We can remove a weight of 1 unit from each side and the hanger will stay balanced. This is the same as subtracting 1 from each side of the equation.



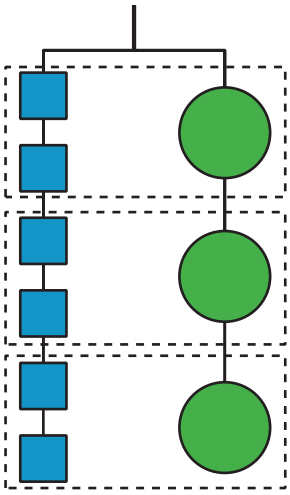
$7 - 1 = 3x + 1 - 1$

An equation for the new balanced hanger is $6 = 3x$.



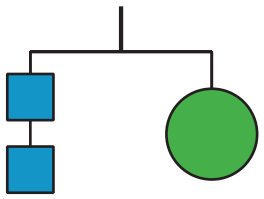
$6 = 3x$

We can make 3 equal groups on each side and the hanger will stay balanced. This is the same as dividing each side of the equation by 3 (or multiplying each side by $\frac{1}{3}$). In other words, the hanger will balance with $\frac{1}{3}$ of the weight on each side.



$\frac{1}{3} \cdot 6 = \frac{1}{3} \cdot 3x$

The two sides of the hanger balance with two 1-unit weights on one side and 1 weight of unknown size on the other side. So, the unknown weight is 2 units.



$2 = x$

Here is a concise way to write the steps above:

$7 = 3x + 1$

$6 = 3x$ after subtracting 1 from each side

$2 = x$ after multiplying each side by $\frac{1}{3}$

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