Unit 8 Family Support Materials

Conditional Probability

In this unit, your student will build on their understanding of probability, including conditional probability. The probability of an event is a number that measures how likely the event is to happen. It can be 0, 1, or any number in between. It is 0 if the event will never happen and 1 if the event must happen. If an event occurs half of the time in the long run, then its probability is 0.5. Conditional probability is the probability that one event occurs under the condition that another event occurs.

Here is an example. The table summarizes the type (medium, large, or extra-large) and condition (no cracked eggs, or one or more cracked eggs) of 50 cartons of eggs at a grocery store.

	medium	large	extra-large	total
one or more eggs cracked	1	3	1	5
no cracked eggs	4	22	19	45
total	5	25	20	50

One carton is selected at random. What is the probability that the carton has no cracked eggs?

The probability is 0.9. This is because 45 cartons have no cracked eggs out of a total of 50 cartons, and $\frac{45}{50} = 0.9$. Students also see this type of question written as $P(\text{no cracked eggs})$, which means "the probability that a randomly selected carton has no cracked eggs." In this case, $P(\text{no cracked eggs}) = 0.9$.

What is the probability that the carton has no cracked eggs under the condition that it is a carton of extra-large eggs? This conditional probability is 0.95. This is because 19 cartons of extra-large eggs had no cracked eggs out of a total of 20 cartons of extra-large eggs, and $\frac{19}{20} = 0.95$. This type of probability is called conditional probability because it is a probability based on the condition of selecting a carton of extra-large eggs. Students see this type of question written as $P(\text{no cracked eggs} \mid \text{extra-large})$, which means "the probability that a randomly selected carton has no cracked eggs under the condition that it is a carton of extra-large eggs." In this case $P(\text{no cracked eggs} \mid \text{extra-large}) = 0.95$.

Here is a task to try with your student:

The table summarizes the position of loaves of bread at the grocery store (bread in the front row or bread not in the front row) and the sell-by date (within five days or more than 5 days away).

A loaf of bread is selected at random.

	sell-by date within 5 days	sell-by date more than 5 days away
bread in the front row	36	14
bread not in the front row	24	76

1. What is the probability that the bread has a sell-by date within 5 days?
2. What is the probability that bread has a sell-by date within 5 days under the condition that the loaf of bread is in the front row?
3. What is $P(\text{sell-by date more than 5 days away} \mid \text{bread not in the front row})$?
4. You are in a rush and want to grab a loaf of bread at this store without looking at the sell-by date. Does grabbing the loaf of bread from the front row give you a better chance of getting a loaf of bread with a sell-by date more than 5 days away than grabbing a loaf of bread that is not in the front row? Use probability to explain your reasoning.

Solution:

1. 0.4 or $\frac{60}{150}$ because there are 60 loaves that have a sell-by date within 5 days ($36 + 24 = 60$) out of the 150 loaves at the store ($36 + 14 + 24 + 76 = 150$).
2. 0.72 or $\frac{36}{50}$ because there are 36 loaves that have a sell-by date within 5 days and that are in the front row out of the 50 loaves in the front row ($36 + 14 = 50$).
3. 0.76 or $\frac{76}{100}$ because there are 76 loaves that have a sell-by date more than 5 days away and that are not in the front row out of 100 loaves that are not in the front row ($24 + 76 = 100$).
4. No, it does not give you a better chance of getting a loaf of bread with a sell-by date more than 5 days away. The probability of getting a loaf of bread with a sell-by date more than 5 days away under the condition that it is in the front row is 0.28 ($1 - 0.72 = 0.28$) compared to a probability of 0.76 ($\frac{76}{24+76} = 0.76$) for a loaf of bread that is not in the front row.