Unit 1, Lesson 15

Building Polygons

Goals

- Generalize (orally) that four side lengths do not determine a unique quadrilateral, but that three side lengths can determine a unique triangle.
- Use manipulatives to show that there is a minimum and maximum length that the third side of a triangle could be, given the other two side lengths.

Learning Targets

- I can show that the 3 side lengths that form a triangle cannot be rearranged to form a different triangle.
- I can show that the 4 side lengths that form a quadrilateral can be rearranged to form different quadrilaterals.
- I can show whether or not 3 side lengths will make a triangle.

Lesson Narrative

The goal of the lesson is for students to make sense of the different shapes that are possible under given constraints about side lengths, including whether only one shape is possible or no shape is possible (MP1). In this lesson, students experiment with constructing triangles given 2 or 3 side lengths. They start by working with cardboard strips and metal fasteners to discover that there are some combinations of lengths that do not make a triangle. Then students move toward using a ruler and compass, seeing that doing so recreates the function of the cardboard strips and metal fasteners more efficiently. The purpose of this transition is to help students move toward a mental understanding that does not depend on physical objects, helping them work toward the understanding that in a triangle the sum of any two sides must be greater than the other side.

Students use repeated reasoning with specific cases to formulate a general rule about which side lengths are possible for triangles (MP8).

Standards

Addressing **7.G.A.2**

Instructional Routines

- MLR7: Compare and Connect
- MLR8: Discussion Supports

Required Materials

Materials to Gather

- Geometry toolkits: Activity 1, Activity 2, Activity 3
- Metal paper fasteners: Activity 2, Activity 3
- Compasses: Activity 3

Materials to Copy

- What Can You Build? Cutouts (1 copy for every 2 students): Activity 2

Required Preparation

Activity 2:

For the digital version of the activity, acquire devices that can run the applet.

Activity 3:

Each group needs 2 sets of strips and fasteners from an earlier lesson. If needed, prepare additional sets of strips and fasteners, including punching holes in the strips.

For the digital version of the activity, acquire devices that can run the applet.

Lesson:

For some activities in this lesson, you will need slips cut from copies of the "What Can You Build?" blackline master. Prepare 1 copy for every 2 students. These slips can be reused from one class to the next. To make the slips sturdier, it is recommended to copy them onto card stock. If card stock is not available, consider gluing each copy to light cardboard, such as a cereal box. Also if possible, copy each set of slips on a different color of paper, so that a stray strip can quickly be put back.

After the slips are cut, punch holes into the endpoints of each segment. A standard hole punch makes holes that are a little larger than needed for the metal paper fasteners, causing the cardboard strips to wiggle around. If possible, find a way to punch holes that are slightly smaller than the size of a standard hole punch.

Put each set of strips in an envelope. Prepare to distribute at least 12 metal paper fasteners (i.e., brass brads) to each group.

Note: If using the digital version of every activity, the strips and fasteners will not be needed.

Student Facing Learning Goals

Let's build shapes.

15.1 Where Is Lin?

Warm-up

5 min

Activity Narrative

The purpose of this warm-up is to remind students that when you have a fixed starting point, all the possible endpoints for a segment of a given length form a circle (centered around the starting point). The context of finding Lin's position in the playground helps make the geometric relationships more concrete for students. Since there are many possible distances between Lin and the swings (but not infinitely many), this activity serves as an introduction to formalizing rules about what lengths can and cannot be used to form a triangle.

Monitor for students who come up with different locations for Lin, as well as for students who recognize that there are many possible locations, and ask them to share during the whole-class discussion.

Teacher Notes for IM 6–8 Math Accelerated v.360
The unit on circles is in the previous course instead of a previous unit, as stated in the *Activity Synthesis*. If students have yet to work with compasses this year, take time during the *Activity Synthesis* to demonstrate their use in preparation for future activities. Make sure students understand that a compass is useful not just for drawing circles, but also for

transferring lengths in general.

Standards

Addressing **7.G.A.2**

Launch

Arrange students in groups of 2. If necessary, remind students of the directions north, south, east, and west and their relative position on a map. Provide access to geometry toolkits. Give students 2 minutes of quiet work time, followed by a partner discussion and a whole-class discussion.

During the partner discussion, have students compare their reasoning with a partner and to discuss until they reach an agreement.

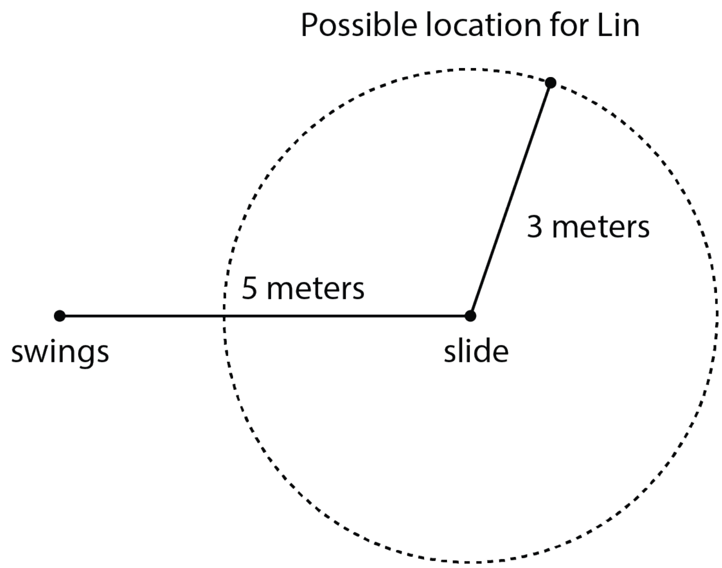
Student Task Statement

At a park, the slide is 5 meters east of the swings. Lin is standing 3 meters away from the slide.

1. Draw a diagram of the situation including a place where Lin could be.
2. How far away from the swings is Lin in your diagram?
3. Where are some other places Lin could be?

Student Response

1. Answers vary. See diagram.



2. There is no way to know for sure, because we don’t know what direction Lin is from the slide. She could be anywhere between 2 and 8 meters away from the swings.
3. Lin could be at any position along a circle that is centered on the slide and that has a radius of 3 meters.

Building on Student Thinking

Some students might assume that the swings, the slide, and Lin are all on a straight line, and that she must be 8 meters away. Ask these students if the problem tells us which direction Lin is from the slide.

Some students may confuse the type of compass discussed in the *Launch* and the type of compass discussed in the *Activity Synthesis*. Consider displaying a sample object or image of each of them and explain that the same name refers to two different tools.

Activity Synthesis

Begin by inviting selected students to share their diagrams of where Lin is located. Discuss the following questions with the whole class:

- "Do we know for sure where Lin is?" (No, because we don't know what direction she is from the swings.)
- "What shape is made by all the possible locations where Lin could be?" (a circle)
- "What is the closest Lin could be to the swings?" (2 m)
- "What is the farthest Lin could be away from the swings?" (8 m)

Consider displaying the applet to show all the locations where Lin could be.

The Geogebra applet 'Where Is Lin?' is available here: https://www.geogebra.org/m/tezEbnJ3.

Based on their work with drawing circles in a previous unit, some students may suggest that a compass could be used to draw all the possible locations where Lin could be. Consider having a student demonstrate how this could be done. If not mentioned by students, it is *not* necessary for the teacher to bring it up at this point.

15.2 Building Diego's and Jada's Shapes

10 min

Activity Narrative

There is a digital version of this activity.

The purpose of this activity is to reinforce that some conditions define a unique polygon while others do not. Students build polygons given only a description of the polygons' side lengths. They articulate that this is not enough information to guarantee that a pair of quadrilaterals are identical copies. On the other hand, triangles have a special property that three specific side lengths result in a unique triangle. Students should notice that their recreation of Jada's triangle is rigid—the side lengths and angles are all fixed.

Monitor for students who try putting the side lengths together in different orders to build different polygons, and invite them to share during the whole-class discussion.

In the digital version of the activity, students use an applet to create polygons dynamically. The digital version may reduce barriers for students who need support with fine-motor skills and for students who benefit from extra processing time.

Standards

Addressing **7.G.A.2**

Instructional Routines

- MLR7: Compare and Connect

Launch

Arrange students in groups of 2. Ensure that each group has one set of strips and fasteners, as well as access to the geometry toolkit, including rulers and protractors. Encourage students to think about whether there are different shapes that would fulfill the given conditions. Give students 5–6 minutes of group work time followed by a whole-class discussion.

Access for Students with Disabilities

Action and Expression: Provide Access for Physical Action. Provide access to tools and assistive technologies such as a device that can run the digital applet.
Supports accessibility for: Visual-Spatial Processing, Conceptual Processing, Organization

Student Task Statement

1. Diego built a quadrilateral using side lengths of 4 in, 5 in, 6 in, and 9 in.
    a. Build such a shape.
    b. Is your shape an identical copy of Diego's shape? Explain your reasoning.
2. Jada built a triangle using side lengths of 4 in, 5 in, and 8 in.
    a. Build such a shape.
    b. Is your shape an identical copy of Jada's shape? Explain your reasoning.

Student Response

1. No, my quadrilateral is probably not an identical copy of Diego's quadrilateral, because it is floppy. There are lots of different angles I can use to make a quadrilateral with these side lengths.
2. Yes, my triangle should be an identical copy of Jada's triangle, because it is not floppy. There is no way to change the angles to make a different triangle with these side lengths.

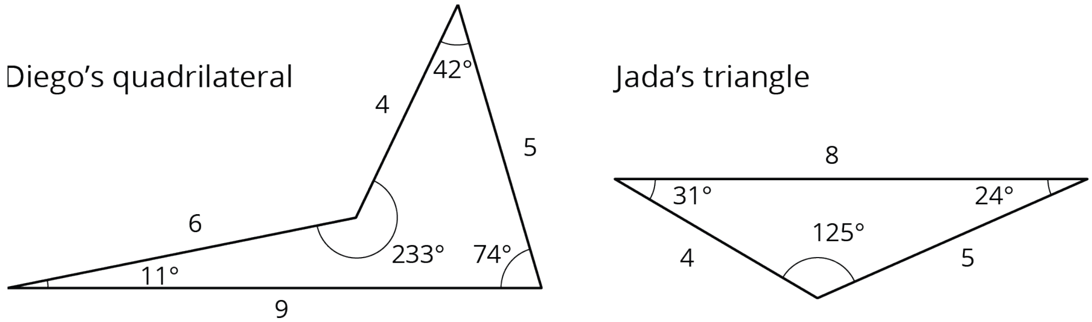
Building on Student Thinking

Students may think that their triangle is different from Jada's because hers is "upside down." Ask the student to turn their triangle around and ask them if it is now a different triangle. While there is a good debate to be had if they continue to insist they are different, let the students know that, for this unit, we will consider shapes that have been turned or flipped or moved to be identical copies and thus "not different."

Activity Synthesis

Select previously identified students to share their constructions and explanations. Display each student's example for all to see.

If desired, reveal Diego and Jada's shapes, and display for all to see alongside students' work.



Ask students:

- "Is this what you thought Jada and Diego's shapes looked like?"
- "Which shape did you make an identical copy of?" (Jada's triangle.)
- "Why did you not make an identical copy of Diego's shape?" (Because you can make quadrilaterals with the same side lengths but different angle measure.)

Access for English Language Learners

MLR7 Compare and Connect. Lead a discussion comparing, contrasting, and connecting the different representations. Display a student's quadrilateral that is different from Diego's and ask, "How are these quadrilaterals the same? How are they different?" "How does the order of the sides impact the quadrilateral?".
Advances: Representing, Conversing

15.3 How Long Is the Third Side?

15 min

Activity Narrative

There is a digital version of this activity.

The purpose of this activity is for students to experience that the sum of the lengths of the two shorter sides of a triangle must be greater than the length of the longest side. Students continue working with the cardboard strips and fasteners from a previous lesson to see how many different triangles they can build given two of the three side lengths. In the *Activity Synthesis*, the possible triangles are arranged in a way that helps students see the structure of an unknown angle between two known side lengths as a hinge (MP7). This prepares students for using compasses to draw triangles with given side lengths. They also continue to work at recognizing when two triangles are identical copies that are oriented differently.

As students work, monitor for those who:

- Find different lengths for the third side of the triangle.
- Use precise language to describe how the two side lengths can move in relation to each other.
- Make a connection to the circle of Lin's possible positions from the previous activity.

In the digital version of the activity, students use an applet to create polygons dynamically. The digital version may reduce barriers for students who need support with fine-motor skills and for students who benefit from extra processing time.

Teacher Notes for IM 6–8 Math Accelerated v.360
This *Activity Narrative* refers to "cardboard strips and fasteners from a previous lesson." In this course, these materials were first prepared in a previous activity, not a previous lesson.

Standards

Addressing **7.G.A.2**

Instructional Routines

- MLR8: Discussion Supports

Launch

Arrange students in groups of 4. Distribute *two* sets of strips and fasteners to each group. Give students 7–10 minutes of group work time, followed by a whole-class discussion.

Access for Students with Disabilities

Action and Expression: Provide Access for Physical Action. Provide access to tools and assistive technologies such as a device that can run the digital applet.
Supports accessibility for: Visual-Spatial Processing, Conceptual Processing, Organization

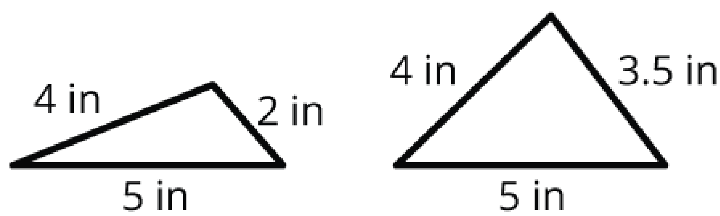
Student Task Statement

Your teacher will give you some strips of different lengths and fasteners that you can use to attach the corners of the strips.

1. Build as many different triangles as you can that have one side length of 5 inches and one of 4 inches. Record the side lengths of each triangle that you build.
2. Are there any other lengths that could be used for the third side of the triangle but weren't in your set?
3. Are there any lengths that were in your set but could not be used as the third side of the triangle?

Student Response

1. There are 5 possible triangles, with the third side measuring 3 inches, 4 inches, 5 inches, 6 inches, or 8 inches.
2. Yes. The third side could be 2 inches, 7 inches, or any fractional length between 1 and 9 inches. Sample responses:



3. Yes. We could not use the 9-inch side to make a triangle, because it is a straight line when we connect it.

Building on Student Thinking

Some students may think that the third side of the triangle cannot be 4 or 5 inches, because then the triangle would have two sides of that length instead of the one asked for in the question. Explain that the triangle is acceptable as long as *at least* one side is 5 inches long and *at least* one side is 4 inches.

If students claim that 8 is the longest the third side can be and that 2 would be the shortest (if they were given a strip of that length), ask them to consider fractional side lengths.

Are You Ready for More?

Assuming you had access to strips of any length, and you used the 9-inch and 5-inch strips as the first two sides, complete the sentences:

1. The third side can't be ________ inches or longer.
2. The third side can't be ________ inches or shorter.

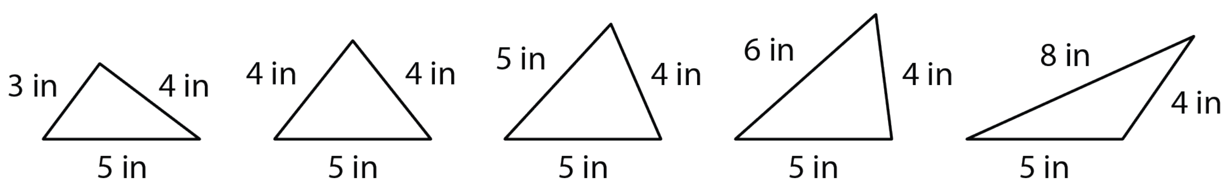
Extension Student Response

The longest the third side could be is shorter than 14 inches (such as 13 or 13.9).

The shortest the third side could be is longer than 4 inches (such as 5 or 4.1).

Activity Synthesis

Select previously identified groups to share a triangle that they created. Establish whether each new triangle shared is the same as a triangle previously shared or is a different triangle. Collect one example of each possible triangle and display them for all to see, in order of increasing side length for the third side. Continue until students agree that all possible triangles are displayed.



To help students generalize about all the possible triangles that could be built with sides of 4 inches and 5 inches, ask questions like the following:

- "What do you notice about the triangles?"
- "Why was it impossible to use the 9-inch side to create a triangle?"
- "What is the longest the third side of the triangle could be?" (more than 8, but less than 9, e.g., 8.5, 8.75, 8.9)
- "What is the shortest the third side of the triangle could be?" (less than 2, but more than 1, e.g., 1.5, 1.25, 1.1)
- "What happens when the third side is 1 inch or 9 inches?" (You get a straight line instead of a triangle.)

Display a 5-inch strip fastened to a 4-inch strip for all to see. Demonstrate rotating the 4-inch strip around 180° to line up with each of the displayed triangles, as well as to show the idea that the third side could have a fractional side length. Invite students to share how this relates to the previous activity about Lin's distance from the swings. (If we hold one strip fixed, then all the possible locations where the other strip could end form a circle.)

Access for English Language Learners

MLR8 Discussion Supports. Display sentence frames to support students as they generalize about the length of the third side of the triangle. Examples: "The third side of a triangle will always be ________ because . . . ." "I agree because . . . ." "I wonder if . . . ."
Advances: Speaking, Conversing

Lesson Synthesis

The purpose of this discussion is for students to consider how given side lengths affect the conditions of polygons they create. Ask students:

- How is building a triangle with three given side lengths different from building a quadrilateral with four given side lengths? (The triangle must be a specific one, but the quadrilateral might be a lot of different things by changing the angles.)

Then display these 3 sets of triangle side lengths for all to see. Ask students which sets could create a triangle, and how they know.

- 1, 1, 5
- 4, 4, 4
- 5, 5, 1

Once students have had time to think about the sets of numbers, display this image and ask which set it would correspond to. (1,1,5) Invite students to share their thinking or informal diagrams about the other side lengths. (4,4,4 is all the same, so it will make an equilateral triangle. 5,5,1 could be tall and skinny, but it will work.)

If time allows, ask students, "If you draw one side of the triangle with circles on each end, and the circles do cross, they will cross twice. Why do we say there's only one possible triangle instead of two?" (The two triangles are identical copies.)

15.4 Finishing Elena's Triangles

5 min

Cool-down

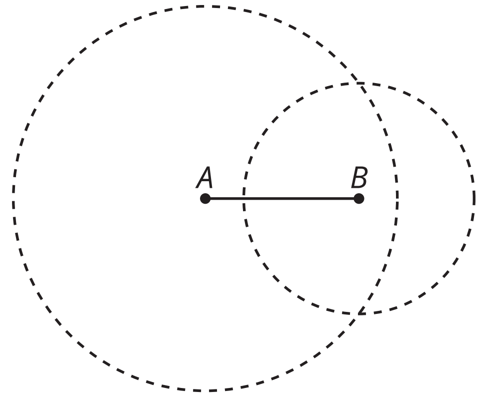
Standards

Addressing **7.G.A.2**

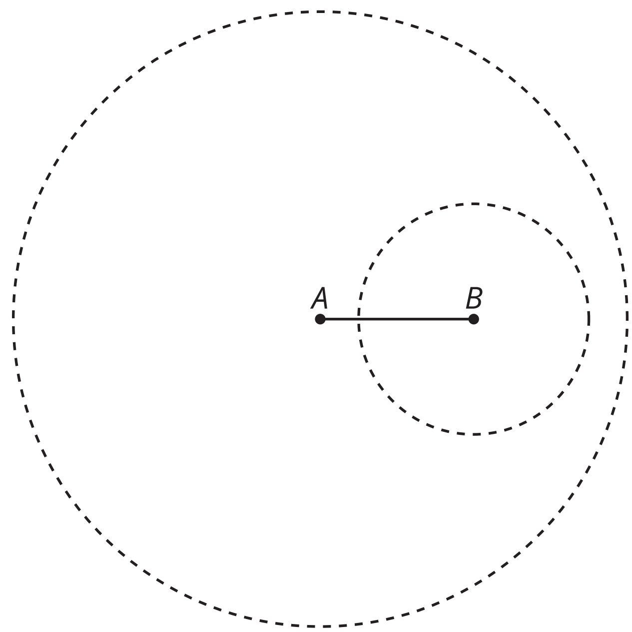
Student Task Statement

1. Elena is trying to draw a triangle with side lengths of 4 inches, 3 inches, and 5 inches.
    - She uses her ruler to draw a 4-inch line segment, AB.
    - She uses her compass to draw a circle around point B with a radius of 3 inches
    - She draws another circle, around point A with a radius of 5 inches.

    What should Elena do next? Explain and show how she can finish drawing the triangle.



2. Now Elena is trying to draw a triangle with side lengths 4 inches, 3 inches, and 8 inches. Explain what Elena's drawing means.



Student Response

1. Elena should put a point where the two circles intersect and draw line segments connecting that point to points A and B to finish her triangle.
2. Elena's drawing means that there is no way to draw a triangle with these side lengths. The circles do not intersect, because the side lengths of 3 inches and 4 inches are too short to make a triangle with the third side of 8 inches.

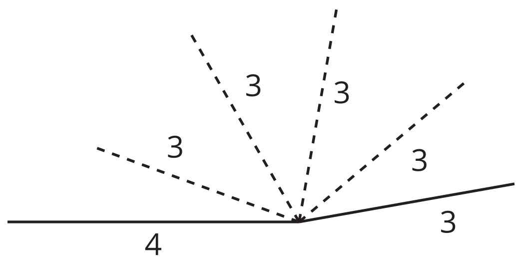
Responding to Student Thinking

Points to Emphasize
If most students struggle with how to finish constructing the triangle, revisit the process of building triangles with side or angle requirements when doing this activity:

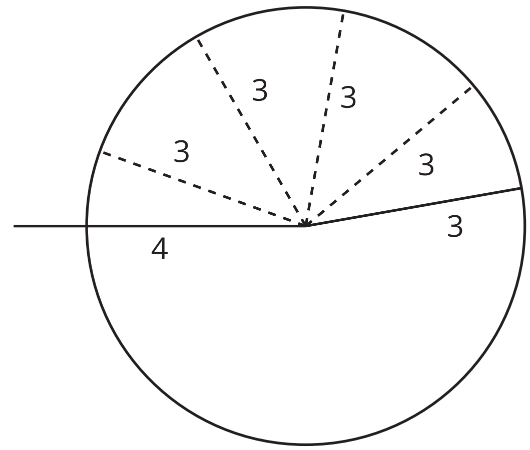
Accelerated 7, Unit 1, Lesson 16, Activity 3 How Many Can You Draw?

Lesson 15 Summary

If we want to build a polygon with two given side lengths that share a vertex, we can think of them as being connected by a hinge that can be opened or closed:



All of the possible positions of the endpoint of the moving side form a circle:

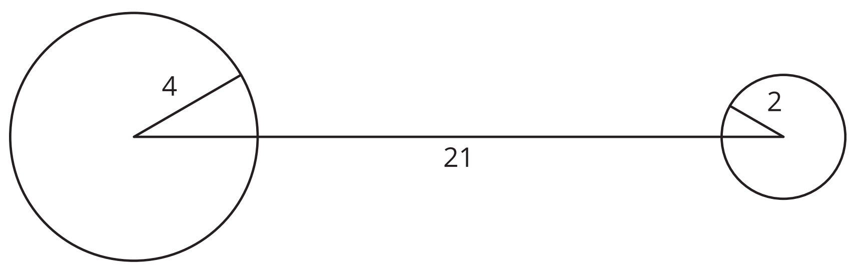


You may have noticed that sometimes it is not possible to build a polygon given a set of lengths. For example, if we have one really, really long segment and a bunch of short segments, we may not be able to connect them all up.

Here's what happens if you try to make a triangle with side lengths 21, 4, and 2:

The short sides don't seem like they can meet up because they are too far away from each other.

If we draw circles of radius 4 and 2 on the endpoints of the side of length 21 to represent positions for the shorter sides, we can see that there are no places for the short sides that would allow them to meet up and form a triangle.



In general, the longest side length must be less than the sum of the other two side lengths. If not, we can't make a triangle!

If we *can* make a triangle with three given side lengths, it turns out that the measures of the corresponding angles will *always* be the same. For example, if two triangles have side lengths 3, 4, and 5, they will have the same corresponding angle measures.

Lesson 15 Practice Problems

1 Student Task Statement

A rectangle has side lengths of 6 units and 3 units. Can you make a different quadrilateral that also has two side lengths of 3 and two side lengths of 6? If so, describe it.

Solution

Yes, you could make a parallelogram or a kite using the side lengths 3, 3, 6, and 6.

2 Student Task Statement

Come up with an example of three side lengths that cannot possibly make a triangle, and explain how you know.

Solution

Sample response: The lengths 1 foot, 1 inch, and 1 inch cannot possibly make a triangle, because if you attach the 1-inch lengths to either end of the 1-foot length, the 1-inch lengths are too short to connect at their other ends.

3 Student Task Statement

Select **all** the sets of three side lengths that will make a triangle.

A. 3, 4, 8

B. 7, 6, 12

C. 5, 11, 13

D. 4, 6, 12

E. 4, 6, 10

Solution

B, C

4 Student Task Statement

Based on signal strength, a person knows their lost phone is exactly 47 feet from the nearest cell tower. The person is currently standing 23 feet from the same cell tower. What is the closest the phone could be to the person? What is the furthest their phone could be from them?

Solution

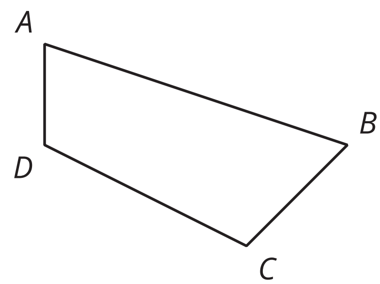
24 feet, 70 feet

5 from Unit 1, Lesson 2

Student Task Statement

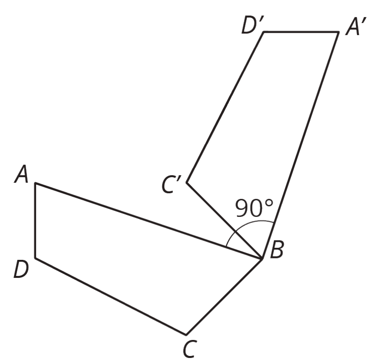
Here is quadrilateral $ABCD$. Draw the image of quadrilateral $ABCD$ after each rotation using B as center.

a. 90° clockwise
b. 120° clockwise
c. 30° counterclockwise



Solution

a.



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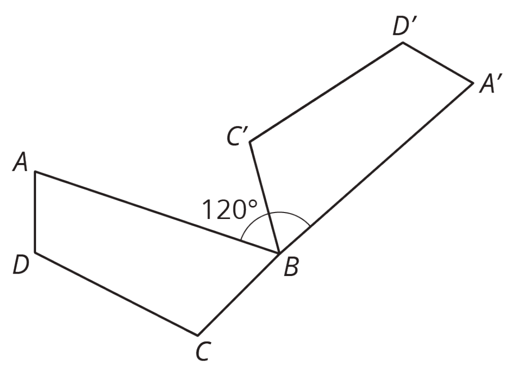
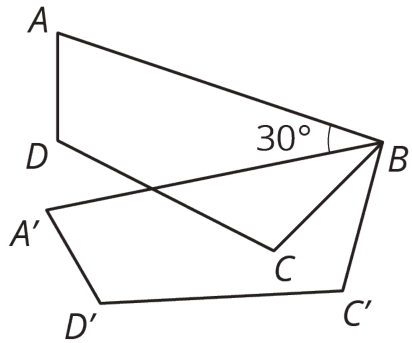


c.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