Unit 9, Lesson 1

Tessellations of the Plane

Goals

- Create and describe patterns with specific polygons that fill the plane.

Lesson Narrative

This optional lesson focuses on the idea of a *tessellation:* a repeating pattern that covers a plane with no gaps or overlaps. The lesson begins with a visual analysis of tessellations where students apply their knowledge of geometry and transformations to describe repeating patterns. In the next activity, students create and compare tessellations by making use of structure (MP7). Finally, students work to describe and recreate tessellations from a partner's description. As they become more precise in their descriptions, their recreations become more exact (MP6). This lesson also includes examples of tessellations used in art from a variety of cultures, which can be explored as an extension.

Standards

Building On	**7.G.A**
Addressing	**8.G.A**

Instructional Routines

- MLR2: Collect and Display
- MLR8: Discussion Supports
- Notice and Wonder
- Take Turns

Required Materials

Materials to Gather

- Tracing paper: Activity 2
- Pre-printed slips, cut from copies of the blackline master: Activity 3

Materials to Copy

- Describing a Tessellation Cards (1 copy for every 4 students): Activity 3

Required Preparation

Activity 2:

For the digital version of the activity, acquire devices that can run the applet.

Student Facing Learning Goals

Let's explore geometric patterns!

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1.1 Notice and Wonder: Polygon Patterns

Warm-up

5 min

Activity Narrative

The purpose of this *Warm-up* is for students to make observations of tessellations, which will be useful when students describe and create their own tessellations in a later activity. While students may notice and wonder many things about these images, tessellations covering the plane with no gaps and the transformations of shapes are the important discussion points.

Studying these patterns and understanding how and why they repeat to fill up the plane is an example of expressing regularity in repeated reasoning (MP8). In this case, the repeated reasoning is continuing to lay out the shapes in the same pattern.

Standards

Building On **7.G.A**

Instructional Routines

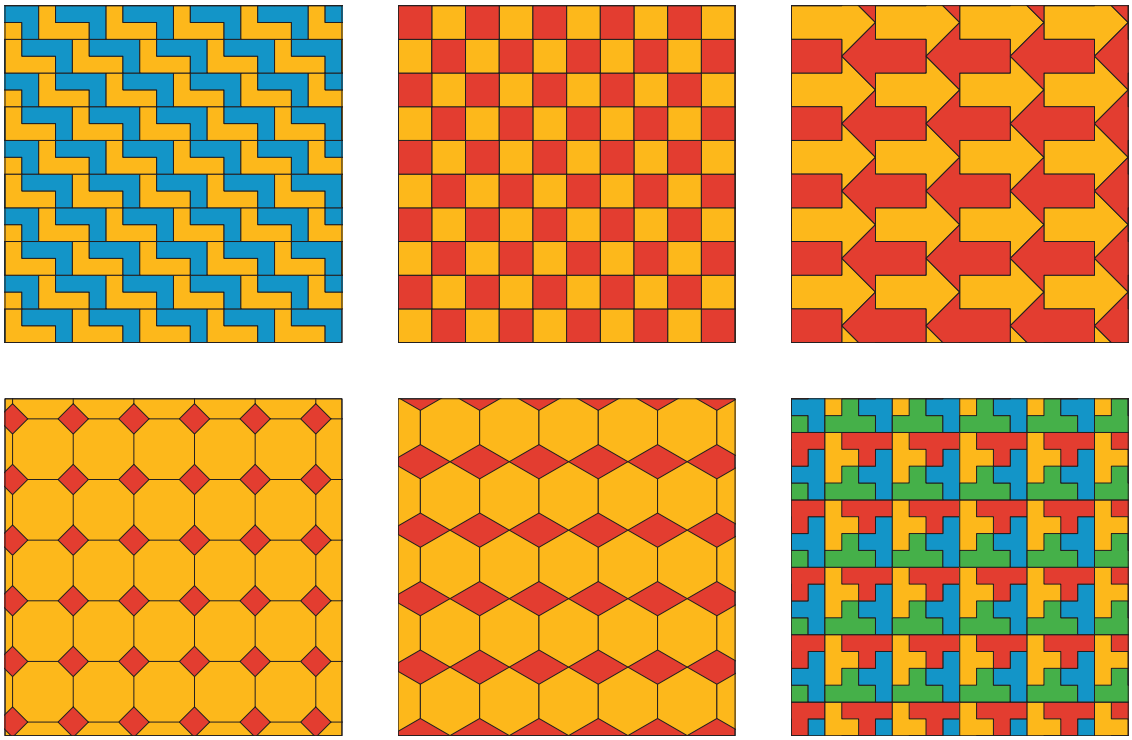
- Notice and Wonder

Launch

Arrange students in groups of 2. Display the images for all to see. Ask students to think of at least one thing they notice and at least one thing they wonder. Give students 1 minute of quiet think time, and then 1 minute to discuss the things they notice and wonder with their partner.

Student Task Statement

What do you notice? What do you wonder?



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Student Response

Students may notice:

- Four of the patterns use only 1 shape. Two of them use 2 different shapes.
- The pattern could continue to the left and right or up and down.

Students may wonder:

- Are the colors of the shapes important?
- Do the arrows count as 1 shape or 2, since they are pointing in different directions?

Activity Synthesis

Ask students to share the things they noticed and wondered. Record and display their responses without editing or commentary for all to see. If possible, record the relevant reasoning on or near the images. Next, ask students, "Is there anything on this list that you are wondering about now?" Encourage students to observe what is on display and respectfully ask for clarification, point out contradicting information, or voice any disagreement.

If the concept of shapes covering the plane without gaps does not come up during the conversation, ask students to discuss this idea.

If time allows, discuss:

- "How do I know what happens as the tessellation continues to grow off of the page?" (There is often a pattern, but we would have to indicate that the pattern continues.)
- "Are the colors of the shapes important?" (They are important for seeing patterns and making the pictures prettier, but they could be changed and the pattern would still be the same.)
- Tessellations of the plane and of spheres have been used by many cultures. Show students pictures of the Kutubiyya Mosque minaret in Marrakesh, Morocco, the Alhambra in Spain, or sphere tessellations such as those found in Japanese temari, and have students describe the patterns and transformations they see.

1.2 Tessellations

20 min

Activity Narrative

There is a digital version of this activity.

The goal of this task is to introduce the notion of a tessellation and then carefully examine how to create a tessellation with each pattern block shape (square, rhombus, equilateral triangle, isosceles trapezoid). For hexagons, there is only one way to fit them together because there will be gaps unless three hexagons meet at each vertex. The other shapes offer much more flexibility, and students have an opportunity to use their artistic creativity.

Students look for and make use of structure (MP7), both when they try to put copies of the shape together to build a tessellation and when they examine whether or not it is possible to construct a different tessellation.

In the digital version of the activity, students use an applet to create tessellations. The applet allows students to select and move shapes. Use the digital version if time or physical materials are limited.

Standards

Building On **7.G.A**

Addressing **8.G.A**

Instructional Routines

- MLR8: Discussion Supports

Launch

Introduce the definition of a **tessellation** of the plane by polygons: a tessellation covers the plane with copies of the shape with no gaps and no overlaps.

Show the students the image from the *Warm-up*, and ask them if these are tessellations. (Yes.)

Demonstrate how to use tracing paper to create a tessellation.

Arrange students in groups of 2. Each pair uses the same shape for their tessellations.

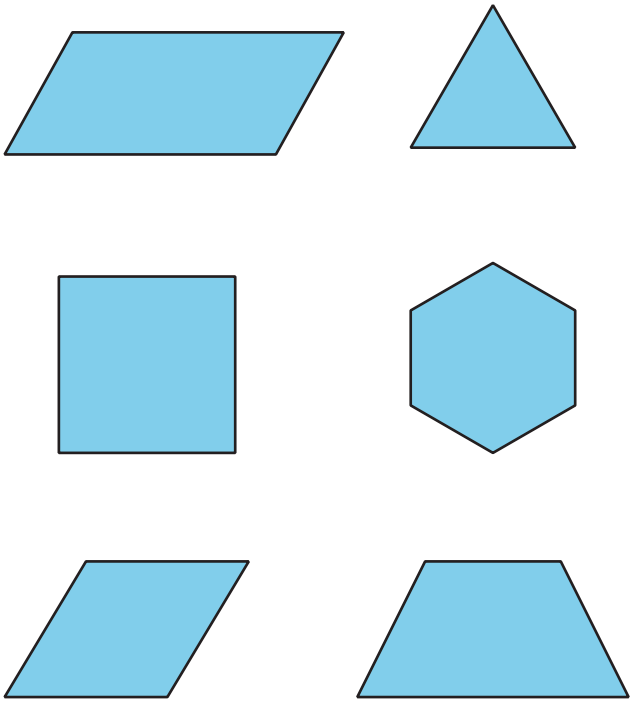
Access for English Language Learners

MLR8 Discussion Supports. Revoice student ideas to demonstrate and amplify mathematical language use. For example, revoice the student statement "The shapes fit together" as "The figures cover the plane edge to edge with no gaps."

Access for Students with Disabilities

Advances: Speaking, Representing Representation: Develop Language and Symbols. Maintain a visible display to record new vocabulary. Invite students to suggest details (words or pictures) that will help them remember the meaning of tessellation and use of tracing paper.
Supports accessibility for: Language, Memory

Student Task Statement



1. Pick one of the shapes. Create a **tessellation** by tracing copies of your shape. Make sure to use the same shape as your partner.
2. Compare your tessellation to your partner's. How are they similar? How are they different?
3. If possible, make a third tessellation (that is different from the ones you and your partner already created) of the plane with your shape. If not possible, explain why it is not possible.

Student Response

1. Answers vary.
2. Answers vary.
3. It depends on the shape. It is not possible for the hexagon: Three hexagons have to come together at each vertex. Once the first hexagon of the pattern is placed, everything else has no flexibility. It is possible for the other shapes: Triangles can be built into hexagons, or they can make rows that can be translated and stacked on top of one another. Parallelograms can also be made into hexagons or rows. Trapezoids can be made into hexagons or rows. Rhombuses can be made into rows and translated. Squares can also be made into rows and translated.

Building on Student Thinking

Tessellations do not need to have symmetric, repeating patterns, though sometimes the shape forces it (as with the regular hexagons). For groups that did not pick the regular hexagon where both partners create tessellations of the "edge to edge" type, that is, the pieces fit together so that only edges of the same length are matched, consider asking:

- "How did you decide to line up your shapes?"
- "How could you line the edges up a different way and still have a tessellation?"

Activity Synthesis

Important questions to address include:

- "Were you able to make different tessellations with your shape?" (It depends on the shape.)
- "If not, why not?" (Three hexagons have to come together at each vertex: once the first hexagon of the pattern is placed, everything else has no flexibility.)
- "If so how?" (Triangles can be built into hexagons, or they can make rows that can be translated and stacked on top of one another. Parallelograms can also be made into hexagons or rows. Trapezoids can be made into hexagons or rows. Rhombuses can be made into rows and translated. Squares can also be made into rows and translated.)
- "What does it look like to not define a tessellation?" (Two octagons can be put together sharing a vertex, but there is a gap that is not large enough for a third octagon.)

Make sure, as students share their ideas for tessellations, to use the language of rigid motions to describe the tessellations. For example, if a student has built a tessellation with parallelograms, choose two parallelograms and ask:

- "How can I use rigid motions to move one of these parallelograms into the position of the other?"
- "Are there other ways I could do this?"

1.3 Describing a Tessellation

20 min

Activity Narrative

In this partner activity, students take turns describing a tessellation of the plane. As students trade roles explaining their thinking and listening, they have opportunities to explain their reasoning and critique the reasoning of others (MP3). While students do not need to use the words "translation," "rotation," or "reflection," their understanding of rigid motions of the plane will play a key role in explaining (and interpreting) where to place each shape in a tessellation. Communicating a geometric pattern clearly in words fully engages students in attending to precision (MP6).

For the *Warm-up*, each student will describe a tessellation while the partner identifies a picture of the tessellation. After a brief discussion of what language was most helpful, students then take turns describing a tessellation as their partner attempts to build the tessellation. They switch roles and then reflect on any misinterpretations that happened and how they may have been related to the language used to describe the tessellations.

Access for English Language Learners

This activity uses the *Collect and Display* math language routine to advance conversing and reading as students clarify, build on, or make connections to mathematical language.

Standards

Building On	**7.G.A**
Addressing	**8.G.A**

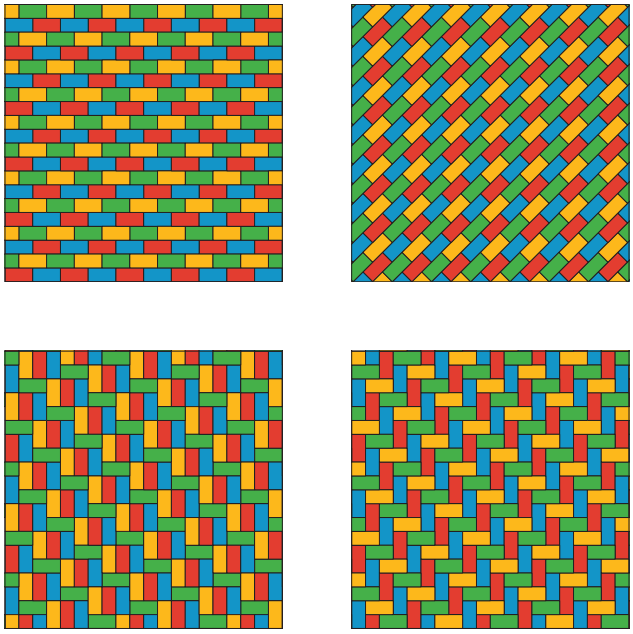
Instructional Routines

- MLR2: Collect and Display
- Take Turns

Launch

Arrange students in groups of 2. Have one partner pick one of the figures and describe the tessellation. Their partner

will find which tessellation they are describing. Then have the partners trade roles.



Use *Collect and Display* to create a shared reference that captures students' developing mathematical language. Collect the language students use to describe the tessellations. Display words such as "rotate," "translate," "reflect," "turn," and "move."

Before discussing their descriptions, direct students' attention to the reference created using *Collect and Display.* Invite students to borrow language from the display as needed and update the reference to include additional phrases as they share their descriptions. (For example, the display may have "Three of the rectangles are next to each other" already on in and be updated with the more precise sentence "Three of the rectangles are aligned vertically while the fourth is horizontal.")
After students have shared their descriptions, discuss:

- "In what ways was describing the tessellation difficult?" (finding words to communicate how the rectangles are aligned with one another)
- "Did you use the words translate, rotate, or reflect?" (Instead of "translate," students may use words like "move." Similarly, students may describe rotations with words like "turn.")
- "What was challenging about describing or identifying the tessellation?"

Distribute a set of 2 cards to each pair of students to complete the activity.

Student Task Statement

1. You and your partner each have a card with a tessellation. Describe what is on your card so that your partner can produce the tessellation. (This should be done so that you cannot see your partner's work until it is complete.)
2. Check together to see if your partner's tessellation agrees with your card and discuss any differences.
3. Change roles so your partner describes a tessellation, which you try to produce.
4. Check the accuracy of your construction and discuss any discrepancies.

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Student Response

Sample response (for Card A): Draw a hexagon. Use each of the hexagon's sides as one side of an equilateral triangle drawn outside the hexagon. The hexagon with its 6 triangles should look like a star. Now we make more stars, with an edge of 1 triangle from each new star sharing 1 side with a triangle on a previous star. If we draw new stars consistently on the same side of existing triangles (in this case, always on the clockwise side), we get the image in Card A.

Sample response (for Card B): Draw a square with horizontal and vertical sides. Use the midpoint of the right side as the top left vertex of a second square the same size. Repeat for the other 3 midpoints of the original square: The midpoint of the top should be the bottom left of a new square, the midpoint of the left should be the bottom right of a new square, and the midpoint of the bottom should be the top right of a new square. To get the figure in Card B, repeat this process for each square arising.

Activity Synthesis

After students recreate their partner's tessellation, discuss as a whole class the use of the words "translate," "reflect," and "rotate" (or equivalents).

- "When you used a translation, did you specify the direction and how far?" (No, for Card B I told my partner to start with a square and then to draw a congruent square so they would share half a side.)
- "When a rotation was involved, did you specify the number of degrees of the rotation?" (Yes, for the rows of equilateral triangles on Card A, I said to rotate 180 degrees about the midpoint of the side of the triangle to draw the next triangle.)
- "When a rotation was involved, did you specify the center of rotation?" (Answers vary)
- "Did you use any reflections?" (Answers vary)

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