Unit 7, Lesson 15

A New Way to Measure Angles

Let's look at a new way to measure angles.

15.1 A One-Unit Radius

A circle has radius 1 unit. Find the length of the arc defined by each of these central angles. Give your answers in terms of π.

1. 180 degrees
2. 45 degrees
3. 270 degrees
4. 225 degrees
5. 360 degrees

15.2 Walking Around

There is a circular path around a pond with a fountain in the center. The path is 2 miles long. When Tyler enters the path, he is east of the fountain. He starts walking counterclockwise.

1. A sign says Tyler has walked half a mile. Sketch an image that shows the path, the fountain, the arc Tyler has walked, and the central angle defined by that arc.

2. Tyler is now northwest of the fountain.

   a. What is the total distance Tyler has walked?

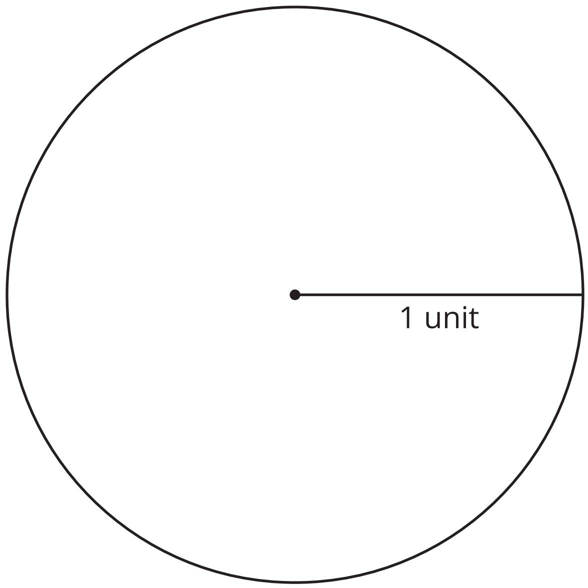
   b. What is the central angle defined by the arc Tyler has walked?

3. There is a duck statue $1\frac{1}{3}$ miles around the path. What is the central angle defined by the arc Tyler has walked when he reaches the statue?

15.3 Defining Radians

Suppose we have a circle that has a central angle. The **radian** measure of the angle is the ratio of the length of the arc defined by the angle to the circle's radius. That is, $\theta = \frac{\text{arc length}}{\text{radius}}$.

1. The image shows a circle with radius 1 unit.



   a. Cut a piece of string that is the length of the radius of this circle.
   b. Use the string to mark an arc on the circle that is the same length as the radius.
   c. Draw the central angle defined by the arc.
   d. Use the definition of a radian to calculate the radian measure of the central angle you drew.

2. Draw a 180-degree central angle (a diameter) in the circle. Use your 1-unit-long piece of string to measure the approximate length of the arc defined by this angle.

3. Calculate the radian measure of the 180-degree angle. Give your answer both in terms of π and as a decimal rounded to the nearest hundredth.

4. Calculate the radian measure of a 360-degree angle.

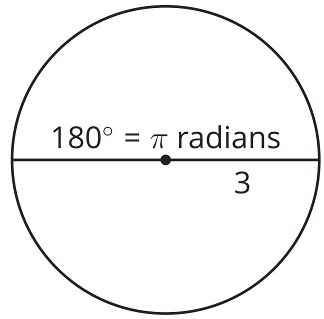
Are you ready for more?

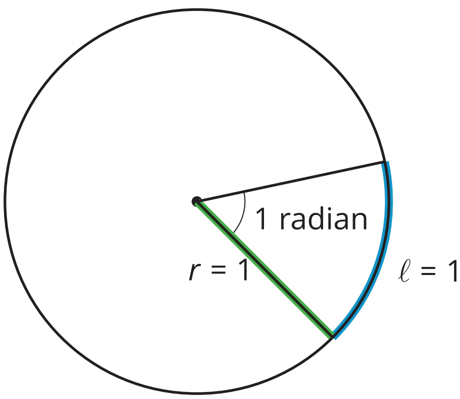
Research where the "360" in 360 degrees comes from. Why did people choose to define a degree as $\frac{1}{360}$ of the circumference of a circle?

Lesson 15 Summary

Degrees are one way to measure the size of an angle. Radians are another way to measure angles. Assume an angle's vertex is the center of a circle. The **radian** measure of the angle is the ratio between the length of the arc defined by the angle and the radius of the circle. We can write this as $\theta = \frac{\text{arc length}}{\text{radius}}$. This ratio is constant for a given angle, no matter the size of the circle.

Consider a 180-degree central angle in a circle with radius 3 units. The arc length defined by the angle is 3π units. The radian measure of the angle is the ratio of the arc length to the radius, which is π radians because $\frac{3\pi}{3} = \pi$.





Another way to think of the radian measure of the angle is that it measures the number of radii that would make up the length of the arc defined by the angle. For example, if we draw an arc that is the same length as the radius, both the arc length and the radius are 1 unit. The radian measure of the central angle that defines this arc is the quotient of those values, or 1 radian.