



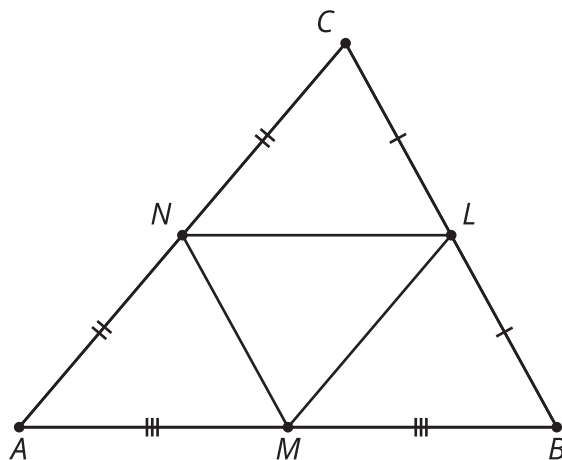
# Splitting Triangle Sides with Dilation (Part 1)

Let's draw segments connecting midpoints of the sides of triangles.

## 5.1 Notice and Wonder: Midpoints

Here's a triangle  $ABC$  with midpoints  $L$ ,  $M$ , and  $N$ . What do you notice? What do you wonder?

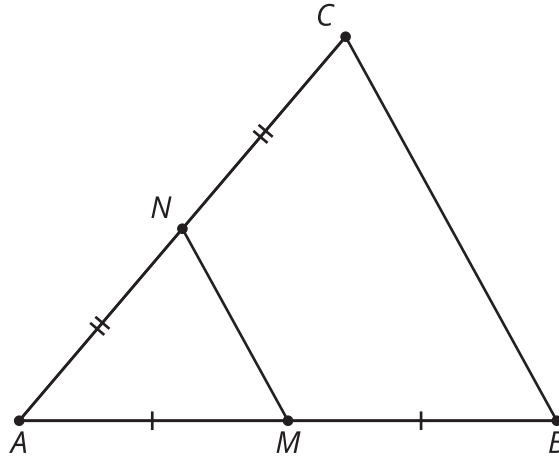
$$\overline{BL} \cong \overline{CL}, \overline{AN} \cong \overline{CN}, \overline{AM} \cong \overline{BM}$$



## 5.2 Dilation or Violation?

Here's a triangle  $ABC$ . Points  $M$  and  $N$  are the midpoints of 2 sides.

$$\overline{AM} \cong \overline{BM}, \overline{AN} \cong \overline{CN}$$



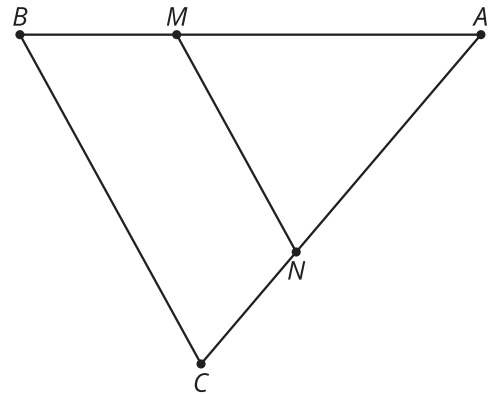
1. Convince yourself that triangle  $ABC$  is a dilation of triangle  $AMN$ . What is the center of the dilation? What is the scale factor?
2. Convince your partner that triangle  $ABC$  is a dilation of triangle  $AMN$ , with the center and scale factor that you found.
3. With your partner, check the definition of dilation on your reference chart, and make sure both of you could convince a skeptic that  $ABC$  definitely fits the definition of dilation.
4. Convince your partner that segment  $BC$  is twice as long as segment  $MN$ .
5. Prove that  $BC = 2 \cdot MN$ . Convince a skeptic.

## 5.3

## A Little Bit Farther Now

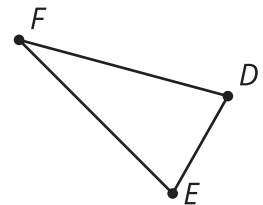
Here's a triangle,  $ABC$ .  $M$  is  $\frac{2}{3}$  of the way from  $A$  to  $B$ .  
 $N$  is  $\frac{2}{3}$  of the way from  $A$  to  $C$ .

What can you say about segment  $MN$ , compared to segment  $BC$ ? Provide a reason for each of your conjectures.



## Are you ready for more?

1. What do you think a negative scale factor could mean? Think about what happens to the point  $E$  after dilating with scale factors  $1, \frac{1}{2}, \frac{1}{4}, \frac{1}{8}$ , and so on getting closer to 0. Then think about what might happen with a zero or negative scale factor.
2. Dilate triangle  $DEF$  using a scale factor of  $-1$  and center  $F$ .
3. How does  $DF$  compare to  $D'F'$ ?
4. Are  $E, F$ , and  $E'$  collinear? Explain or show your reasoning.

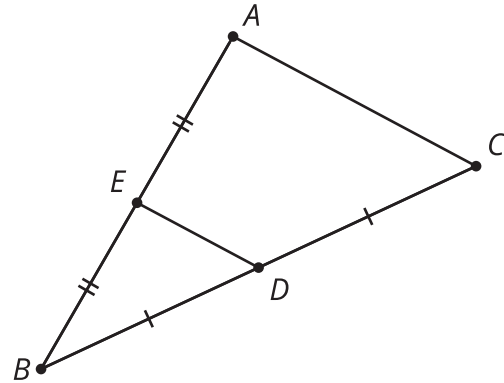


## Lesson 5 Summary

Let's examine a segment whose endpoints are the midpoints of 2 sides of the triangle. If  $D$  is the midpoint of segment  $BC$  and  $E$  is the midpoint of segment  $BA$ , then what can we say about  $ED$  and triangle  $ABC$ ?

Segment  $ED$  is parallel to the third side of the triangle and half the length of the third side of the triangle. For example, if  $AC = 10$ , then  $ED = 5$ . This happens because the entire triangle  $EBD$  is a dilation of triangle  $ABC$ , with a scale factor of  $\frac{1}{2}$ .

$$\overline{BD} \cong \overline{DC}, \overline{BE} \cong \overline{EA}$$



In triangle  $ABC$ , segment  $FG$  divides segments  $AB$  and  $CB$  proportionally. In other words,  $\frac{BG}{GA} = \frac{BF}{FC}$ . Again, there is a dilation that takes triangle  $ABC$  to triangle  $GBF$ , so  $GF$  is parallel to  $AC$ , and we can calculate its length using the same scale factor.

$$\overline{FG} \parallel \overline{AC}$$

