Unit 1, Lesson 21

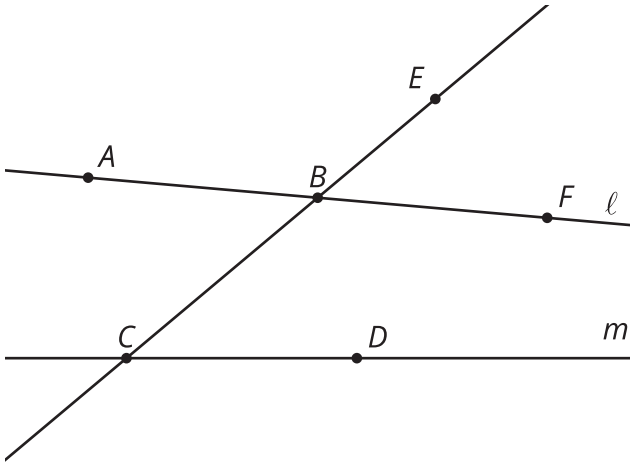
One Hundred Eighty

Let's prove the Triangle Angle Sum Theorem.

21.1 What Went Wrong?

Here are 2 lines ℓ and m that are *not* parallel and have been cut by a transversal.

Tyler thinks angle EBF is congruent to angle BCD because they are corresponding angles and a translation along the directed line segment from B to C would take one angle onto the other. Here are his reasons.



- The translation takes B onto C, so the image of B is C.
- The translation takes E somewhere on ray CB because it would need to be translated by a distance greater than BC to land on the other side of C.
- The image of F has to land somewhere on line m because translations take lines to parallel lines and line m is the only line parallel to ℓ that goes through C.
- The image of F, call it F', has to land on the right side of line BC or else line FF' wouldn't be parallel to the directed line segment from B to C.

1. Your teacher will assign you one of Tyler's statements to think about. Is the statement true? If not, explain your reasoning.

2. In what circumstances are corresponding angles congruent? Be prepared to share your reasoning.

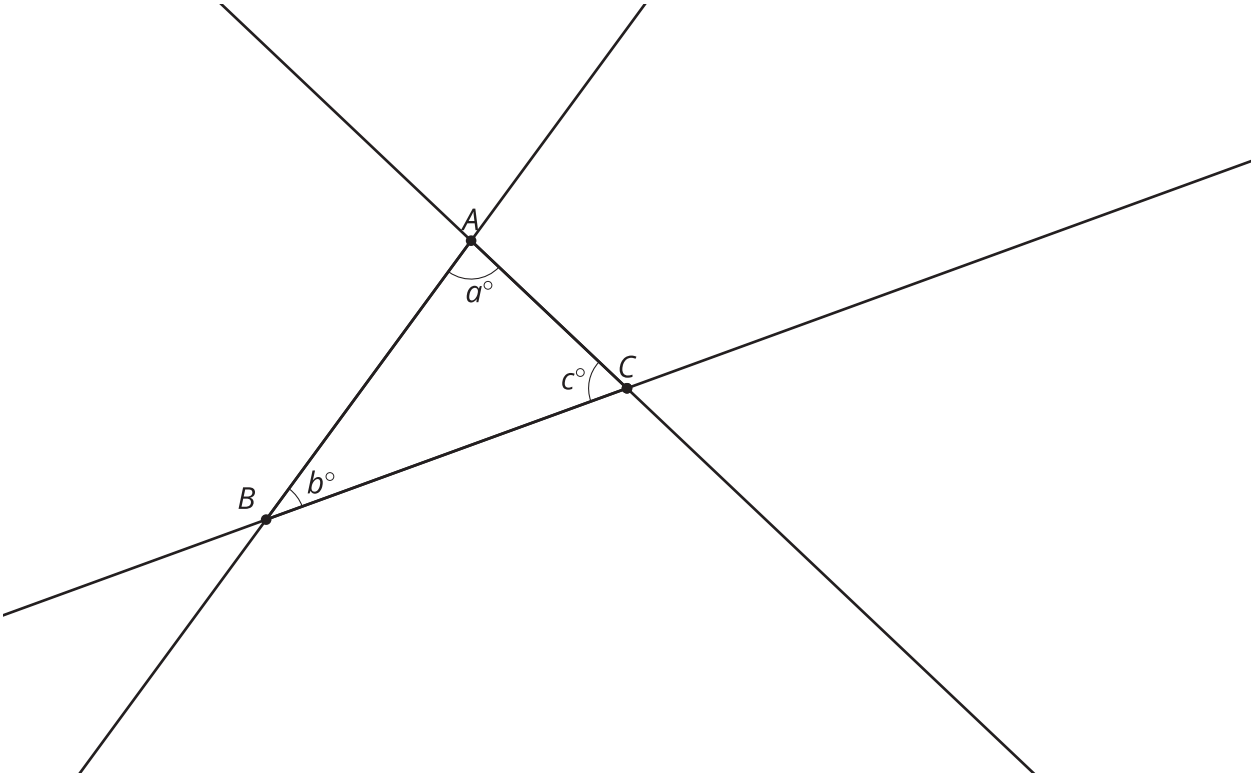
21.2 Triangle Angle Sum One Way

1. Follow these steps to create an image:
   - Use a straightedge to create a triangle. Label the three angle measures as $a°$, $b°$, and $c°$.
   - Extend one side of the triangle in both directions to make a line.
   - Sketch a line parallel to the line you made that goes through the opposite vertex.

2. Use this image to explain why $a + b + c = 180$.

21.3 Triangle Angle Sum Another Way

Here is triangle ABC with angle measures $a°$, $b°$, and $c°$. Each side has been extended to a line.



1. Translate triangle ABC along the directed line segment from B to C to make triangle $A'B'C'$. Label the measures of the angles in triangle $A'B'C'$.
2. Translate triangle $A'B'C'$ along the directed line segment from A' to C to make triangle $A''B''C''$. Label the measures of the angles in triangle $A''B''C''$.
3. Label the measures of the angles that meet at point C. Explain your reasoning.

4. What is the value of $a + b + c$? Explain your reasoning.

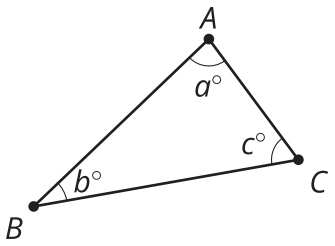
Are you ready for more?

One reason mathematicians like to have rigorous proofs even when conjectures seem to be true is that it can help reveal what assertions were used. This proof is based on rigid transformations that take lines to parallel lines. If our assumptions about parallel lines changed, so would the consequences about triangle angle sums.

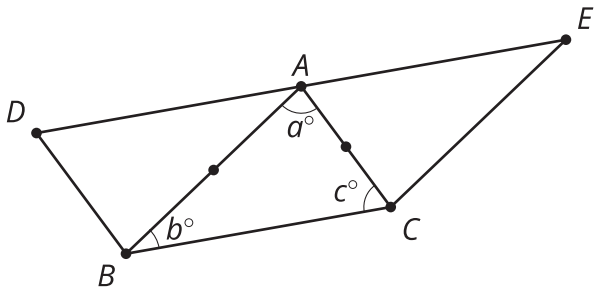
Any study of geometry where these assumptions change is called non-Euclidean geometry. In some non-Euclidean geometries, lines in the same direction may intersect, while in others they do not. In spherical geometry, which studies curved surfaces like the surface of Earth, lines in the same direction always intersect. Imagine a triangle connecting the north pole, a point on the equator, and a second point on the equator one quarter of the way around Earth from the first. What is the sum of the angles in this triangle?

Lesson 21 Summary

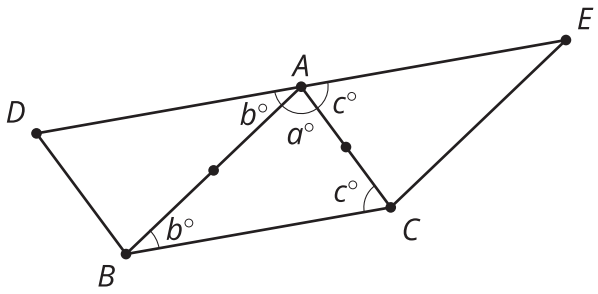
Using rotations and parallel lines, we can understand why the angles in a triangle always add to 180 degrees. Here is triangle ABC.



Rotate triangle ABC 180 degrees around the midpoint of segment AB, and label the image of C as D. Then rotate triangle ABC 180 degrees around the midpoint of segment AC, and label the image of B as E.



Note that each 180-degree rotation takes line BC to a parallel line. So line DA is parallel to BC, and line AE is also parallel to BC. There is only one line parallel to BC that goes through point A, so lines DA and AE are the same line. Since line DE is parallel to line BC, we know that alternate interior angles are congruent. That means that angle BAD also measures $b°$, and angle CAE also measures $c°$.



Since DE is a line, the 3 angle measures at point A must sum to 180 degrees. So $a + b + c = 180$. This argument does not depend on the triangle we started with, so that proves the sum of the 3 angle measures of *any* triangle is always 180 degrees.