

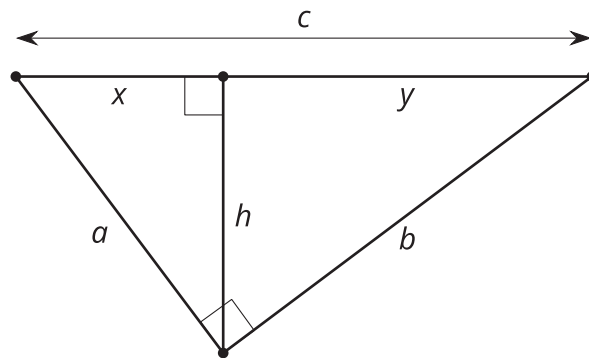


Proving the Pythagorean Theorem

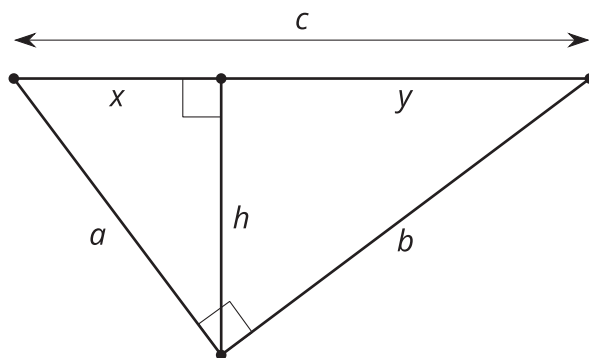
Let's prove the Pythagorean Theorem.

16.1 Notice and Wonder: Variable Version

What do you notice? What do you wonder?



16.2 Prove Pythagoras Right

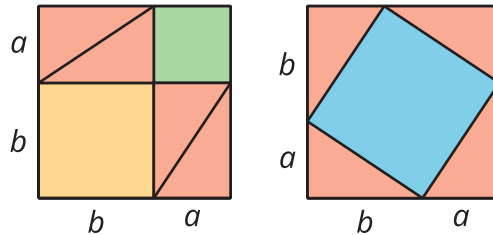


Elena is playing with the equivalent ratios she wrote using this diagram. She rewrites $\frac{a}{x} = \frac{c}{a}$ as $a^2 = xc$. Diego notices and comments, "I got $b^2 = yc$. The a^2 and b^2 remind me of the Pythagorean Theorem." Elena says, "The Pythagorean Theorem says that $a^2 + b^2 = c^2$. I bet we could figure out how to show that."

1. How did Elena get from $\frac{a}{x} = \frac{c}{a}$ to $a^2 = xc$?
2. What equivalent ratios of side lengths did Diego use to get $b^2 = yc$?
3. Prove $a^2 + b^2 = c^2$ in a right triangle with legs length a and b and hypotenuse length c .

16.3

An Alternate Approach



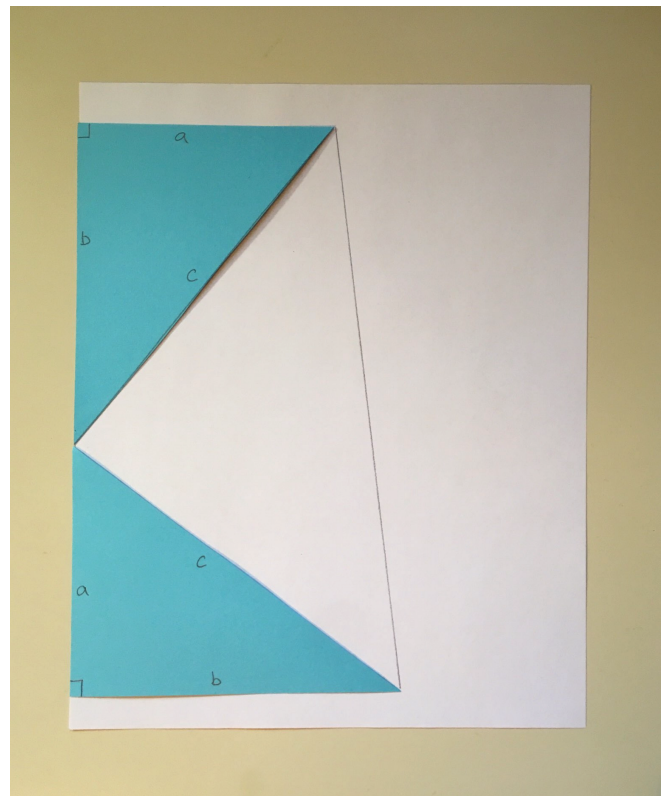
When Pythagoras proved his theorem, he used the two images shown here. Can you figure out how he used these diagrams to prove that $a^2 + b^2 = c^2$ in a right triangle with a hypotenuse of length c ?

💡 Are you ready for more?

James Garfield, the 20th president, proved the Pythagorean Theorem in a different way.

- Cut out 2 congruent right triangles
- Label the long sides b , the short sides a and the hypotenuses c .
- Align the triangles on a piece of paper, with one long side and one short side in a line. Draw the line connecting the other acute angles.

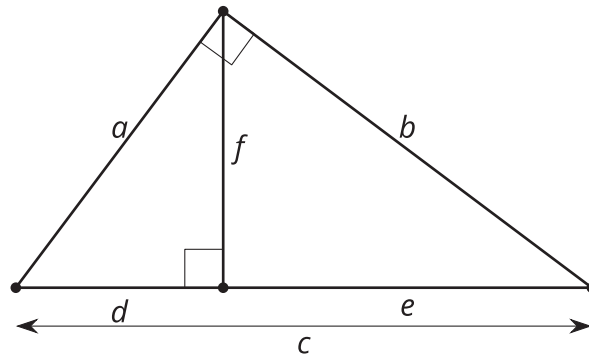
How does this diagram prove the Pythagorean Theorem?



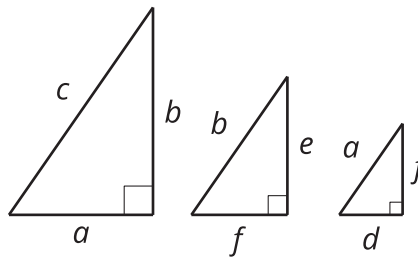
Lesson 16 Summary

In any right triangle with legs a and b and hypotenuse c , we know that $a^2 + b^2 = c^2$. We call this the Pythagorean Theorem. But why does it work?

We can use an altitude drawn to the hypotenuse of a right triangle to prove the Pythagorean Theorem.



We can use the Angle-Angle Triangle Similarity Theorem to show that all 3 triangles are similar. Because the triangles are similar, the corresponding side lengths are in the same proportion.



Because the largest triangle is similar to the smaller triangle, $\frac{c}{a} = \frac{a}{d}$. Because the largest triangle is similar to the middle triangle, $\frac{c}{b} = \frac{b}{e}$. We can rewrite these equations as $a^2 = cd$ and $b^2 = ce$.

We can add the 2 equations to get that $a^2 + b^2 = cd + ce$, or $a^2 + b^2 = c(d + e)$. From the original diagram we can see that $d + e = c$, so $a^2 + b^2 = c(c)$, or $a^2 + b^2 = c^2$.