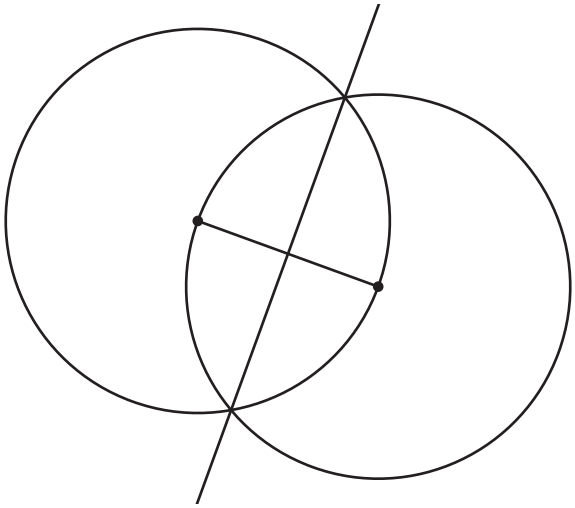
# Bisect It

Let's prove that some constructions we conjectured about really work.

## 14.1 Why Does This Construction Work?



If you are Partner A, explain to your partner what steps were taken to construct the perpendicular bisector in this image.

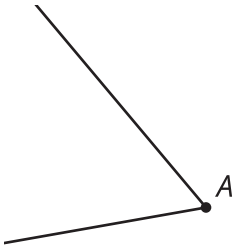
If you are Partner B, listen to your partner's explanation, and then explain to your partner why these steps produce a line with the properties of a perpendicular bisector.

Then work together to make sure the main steps in Partner A's explanation have a reason from Partner B's explanation.

## 14.2 Construction from Definition (Part 1)

Han, Clare, and Andre were given the following task: "Construct an angle bisector. Write a proof that the ray you constructed is the angle bisector of angle $A$."

Read the script your teacher will give you. After each sentence, decide if there is anything to add to the diagram.

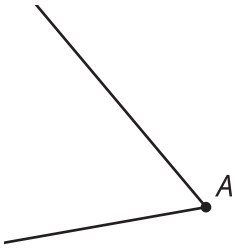



With your group, discuss each student's approach. For each approach, answer these questions:

- What do you notice that this student understands about the problem?
- What question would you ask them to help them move forward?

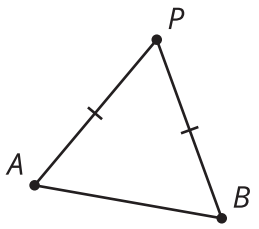
## 14.3 Construction from Definition (Part 2)

Construct an angle bisector. Write a proof that the ray you constructed is the angle bisector of angle $A$.



# 14.4 Reflecting on Reflection

1. Here is a diagram of an isosceles triangle $APB$ with segment $AP$ congruent to segment $BP$.



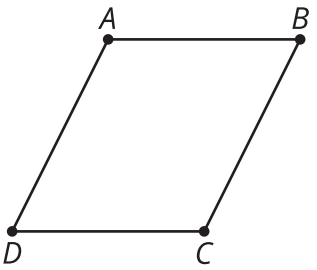

Here is a valid proof that the angle bisector of the vertex angle of an isosceles triangle is a line of symmetry.

a. Read the proof, and annotate the diagram with each piece of information in the proof.

b. Write a summary of how this proof shows the angle bisector of the vertex angle of an isosceles triangle is a line of symmetry.

- Segment $AP$ is congruent to segment $BP$ because triangle $APB$ is isosceles.
- The angle bisector of $APB$ intersects segment $AB$. Call that point $Q$.
- By the definition of angle bisector, angles $APQ$ and $BPQ$ are congruent.
- Segment $PQ$ is congruent to itself.
- By the Side-Angle-Side Triangle Congruence Theorem, triangle $APQ$ must be congruent to triangle $BPQ$.
- Therefore, the corresponding segments $AQ$ and $BQ$ are congruent, and corresponding angles $AQP$ and $BQP$ are congruent.
- Since angles $AQP$ and $BQP$ are both congruent and supplementary angles, each angle must be a right angle.
- So $PQ$ must be the perpendicular bisector of segment $AB$.
- Because reflection across perpendicular bisectors takes segments onto themselves and swaps the endpoints, when we reflect the triangle across $PQ$, the vertex $P$ will stay in the same spot, and the 2 endpoints of the base, $A$ and $B$, will switch places.
- Therefore, the angle bisector $PQ$ is a line of symmetry for triangle $APB$.

2. Here is a diagram of parallelogram $ABCD$.




Here is an invalid proof that a diagonal of a parallelogram is a line of symmetry.

a. Read the proof, and annotate the diagram with each piece of information in the proof.

b. Find the errors that make this proof invalid. Highlight any lines that have errors or false assumptions.

- The diagonals of a parallelogram intersect. Call that point $M$.
- The diagonals of a parallelogram bisect each other, so $MB$ is congruent to $MD$.
- By the definition of parallelogram, the opposite sides $AB$ and $CD$ are parallel.
- Angles $ABM$ and $ADM$ are alternate interior angles of parallel lines, so they must be congruent.
- Segment $AM$ is congruent to itself.
- By the Side-Angle-Side Triangle Congruence Theorem, triangle $ABM$ is congruent to triangle $ADM$.
- Therefore, the corresponding angles $AMB$ and $AMD$ are congruent.
- Since angles $AMB$ and $AMD$ are both congruent and supplementary angles, each angle must be a right angle.
- So $AC$ must be the perpendicular bisector of segment $BD$.
- Because reflection across perpendicular bisectors takes segments onto themselves and swaps the endpoints, when we reflect the parallelogram across $AC$, the vertices $A$ and $C$ will stay in the same spot, and the 2 endpoints of the other diagonal, $B$ and $D$, will switch places.
- Therefore, diagonal $AC$ is a line of symmetry for parallelogram $ABCD$.

## Are you ready for more?

There are quadrilaterals for which the diagonals are lines of symmetry.

1. What is an example of such a quadrilateral?
2. How would you modify this proof to be a valid proof for that type of quadrilateral?

## Lesson 14 Summary

Earlier we constructed an angle bisector, but we did not prove that the construction always works. Now that we know more, we can see why each step is necessary for the construction to precisely bisect an angle. The proof uses some ideas from constructions:

- The midpoint of a segment divides the segment into 2 congruent segments.
- All the radii of a given circle are congruent.

But it also uses some ideas from triangle congruence:

- If triangles have 2 pairs of sides and the angle between them congruent, then the triangles are congruent.
- If triangles are congruent, then the corresponding parts of those triangles are also congruent.

Triangle congruence theorems and properties of rigid transformations can be useful for proving many things, including constructions.