## Lesson 12: Polynomial Division (Part 1)

* Let’s learn a way to divide polynomials.

### 12.1: Notice and Wonder: A Different Use for Diagrams

What do you notice? What do you wonder?

A.

|  |  | 5 |
| --- | --- | --- |
|  |  |  |
| -3 |  | -15 |

B.

|  |  |  | -4 |
| --- | --- | --- | --- |
|  |  |  |  |
| -1 |  |  | +4 |

C.

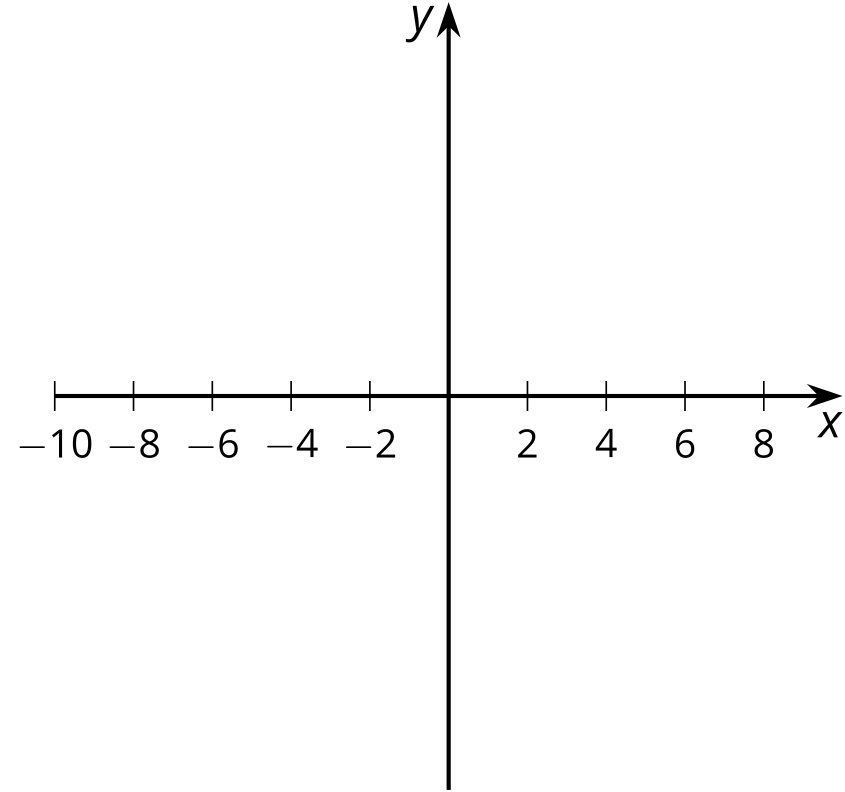
|  |  |  |  |
| --- | --- | --- | --- |
|  |  |  |  |
| -2 |  |  |  |

### 12.2: Factoring with Diagrams

Priya wants to sketch a graph of the polynomial defined by . She knows , so she suspects that could be a factor of and writes  and draws a diagram.

|  |  |  |  |
| --- | --- | --- | --- |
|  |  |  |  |
| -1 |  |  |  |

1. Finish Priya’s diagram.
2. Write as the product of and another factor.
3. Write as the product of three linear factors.
4. Make a sketch of .



### 12.3: More Factoring with Diagrams

Here are some polynomial functions with known factors. Rewrite each polynomial as a product of linear factors. Note: you may not need to use all the columns in each diagram. For some problems, you may need to make another diagram.

1. ,

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
|  |  | * 0 |  |  |  |
| * -7 |  |  |  |  |  |

1. ,

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
|  |  |  |  |  |  |
|  |  |  |  |  |  |

1. ,

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
|  |  |  |  |  |  |
| * 3 |  |  |  |  |  |

1. , ,

* (Hint: )

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
|  |  |  |  |  |  |
|  |  |  |  |  |  |

1. , , ,

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
|  |  |  |  |  |  |
|  |  |  |  |  |  |

#### Are you ready for more?

A diagram can also be used to divide polynomials even when a factor is not linear. Suppose we know is a factor of . We could write . Make a diagram and find the missing factor.

### Lesson 12 Summary

What are some things that could be true about the polynomial function defined by if we know ? If we think about the graph of the polynomial, the point must be on the graph as a horizontal intercept. If we think about the expression written in factored form, could be one of the factors, since when . How can we figure out whether actually is a factor?

Well, if we assume is a factor, there is some other polynomial where , , and are real numbers and . (Can you see why has to have a degree of 2?) In the past, we have done things like expand to find . Since we already know the expression for , we can instead work out the values of , , and by thinking through the calculation.

One way to organize our thinking is to use a diagram. We first fill in and the leading term of , . From this start, we see the leading term of must be , meaning , since .

|  |  |  |  |
| --- | --- | --- | --- |
|  |  |  |  |
| +2 |  |  |  |

We then fill in the rest of the diagram using similar thinking and paying close attention to the signs of each term. For example, we put in a in the bottom left cell because that’s the product of and . But that means we need to have a in the middle cell of the middle row, since that’s the only other place we will get an term, and we need to get once all the terms are collected. Continuing in this way, we get the completed table:

|  |  |  | +12 |
| --- | --- | --- | --- |
|  |  |  |  |
| +2 |  |  | +24 |

Collecting all the terms in the interior of the diagram, we see that , so . Notice that the 24 in the bottom right was exactly what we needed, and it’s how we know that is a factor of . In a future lesson, we will see why this happened. With a bit more factoring, we can say that .



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