

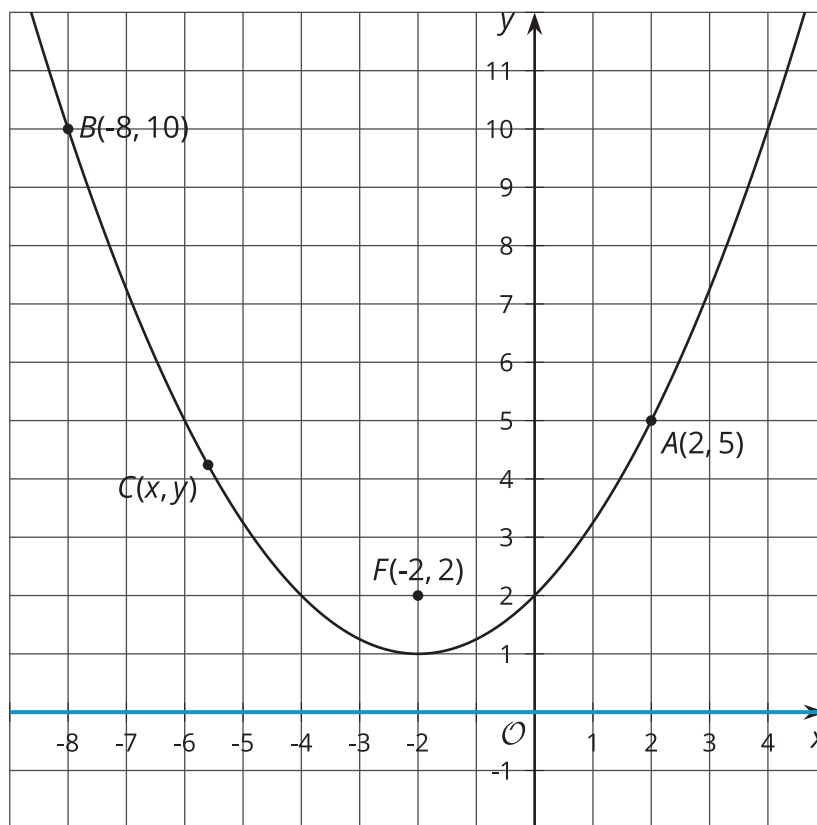


# Equations and Graphs

Let's write an equation for a parabola.

## 19.1 Focus on Distance

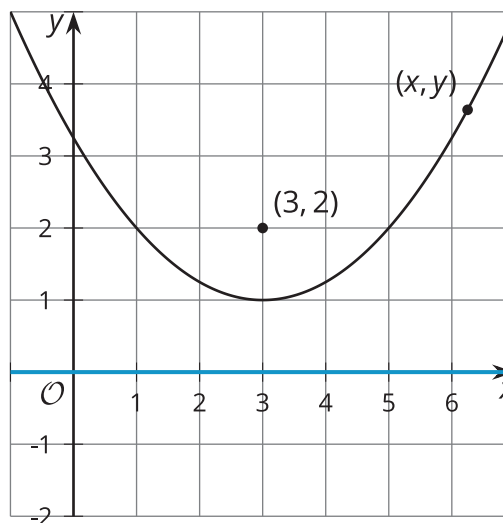
The image shows a parabola with focus  $(-2, 2)$  and directrix  $y = 0$  (the  $x$ -axis). Points  $A$ ,  $B$ , and  $C$  are on the parabola.



Without using the Pythagorean Theorem, find the distance from each plotted point to the parabola's focus. Explain your reasoning.

## 19.2 Building an Equation for a Parabola

The image shows a parabola with focus  $(3, 2)$  and directrix  $y = 0$  (the  $x$ -axis).



1. Write an equation that would allow you to test whether a particular point  $(x, y)$  is on the parabola.
2. The equation you wrote defines the parabola, but it's not in a very easy-to-read form. Rewrite the equation to be in vertex form:  $y = a(x - h)^2 + k$ , where  $(h, k)$  is the vertex.

## 19.3 Card Sort: Parabolas

Your teacher will give you a set of cards. Take turns with your partner to match a graph with an equation.

1. For each match that you find, explain to your partner how you know it's a match.
2. For each match that your partner finds, listen carefully to the explanation. If you disagree, discuss your thinking and work to reach an agreement.

### Are you ready for more?

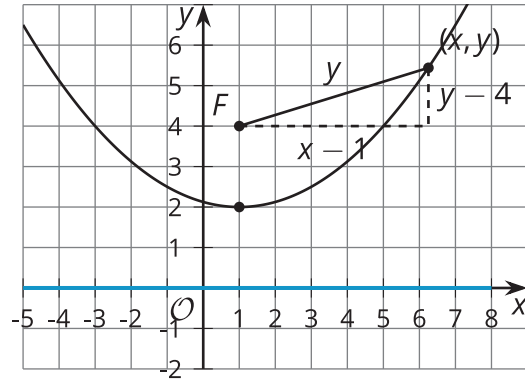
In this section, you have examined points that are equidistant from a given point and a given line. Now consider a set of points that are half as far from a point as they are from a line.

1. Write an equation that describes the set of all points that are  $\frac{1}{2}$  as far from the point  $(5, 3)$  as they are from the  $x$ -axis.
2. Use technology to graph your equation. Sketch the graph and describe what it looks like.

## Lesson 19 Summary

The parabola in the image consists of all the points that are the same distance from the point  $(1, 4)$  as they are from the line  $y = 0$ .

Suppose we want to write an equation for the parabola—that is, an equation that says a given point  $(x, y)$  is on the curve. We can draw a right triangle whose hypotenuse is the distance between point  $(x, y)$  and the focus,  $(1, 4)$ .



The distance from  $(x, y)$  to the directrix, or the line  $y = 0$ , is  $y$  units. By definition, the distance from  $(x, y)$  to the focus must be equal to the distance from the point to the directrix. So, the distance from  $(x, y)$  to the focus can be labeled with  $y$ . To find the lengths of the legs of the right triangle, subtract the corresponding coordinates of the point  $(x, y)$  and the focus,  $(1, 4)$ .

Substitute the expressions for the side lengths into the Pythagorean Theorem to get an equation defining the parabola.

$$(x - 1)^2 + (y - 4)^2 = y^2$$

To get the equation looking more familiar, rewrite it in vertex form, or  $y = a(x - h)^2 + k$  where  $(h, k)$  is the vertex.

$$(x - 1)^2 + (y - 4)^2 = y^2$$

$$(x - 1)^2 + y^2 - 8y + 16 = y^2$$

$$(x - 1)^2 - 8y + 16 = 0$$

$$-8y = -(x - 1)^2 - 16$$

$$y = \frac{1}{8}(x - 1)^2 + 2$$