Unit 7, Lesson 14

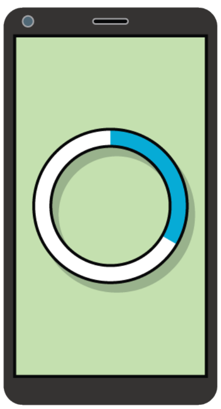
Angles, Arcs, and Radii

Let's analyze relationships between arc lengths, radii, and central angles.

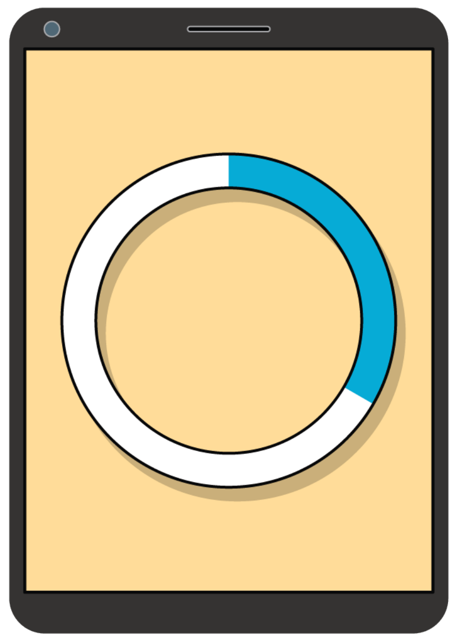
14.1 Comparing Progress

Han and Tyler are each completing the same set of tasks on an online homework site. Han is using his smartphone, and Tyler is using his tablet. Each student's progress is indicated by the circular bar shown in an image. The shaded arc represents the tasks that have been completed. Once a student has finished all the tasks, the full circumference of the circle will be shaded.

Han's progress



Tyler's progress



Tyler says, "The arc length on my progress bar measures 4.75 centimeters. The arc length on Han's progress bar measures 2.25 centimeters. So I've completed more tasks than Han has."

1. Do you agree with Tyler? Why or why not?
2. What information would you need to make a completely accurate comparison between the two students' progress?

14.2 Card Sort: Angles, Arcs, and Radii

Your teacher will give you a set of cards. Each card contains a circle diagram or measurements.

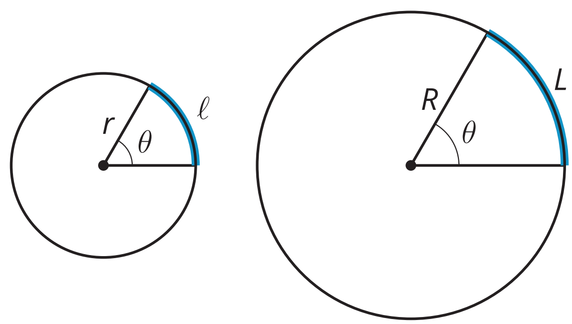
Sort the cards into two groups, one for each diagram. Be prepared to explain how you know each measurement card matches the diagram.

Are you ready for more?

For a circle of radius r, an expression that relates the area of a sector to the arc length defined by that sector is $A = \frac{1}{2}r\ell$, where A is the area of the sector and ℓ is the length of the arc. Explain why this is true, and provide an example.

14.3 A Constant Ratio

Diego and Lin are writing a proof, using these two circles.



Diego says, "We need to prove that, for a given central angle, the arc length is proportional to the radius. That is, the ratio $\frac{\ell}{r}$ has the same value as the ratio $\frac{L}{R}$ because they have the same central angle measure."

Lin says, "The big circle is a dilation of the small circle. If k is the scale factor, then $R = kr$."

Diego says, "The arc length in the small circle is $\ell = \frac{\theta}{360} \cdot 2\pi r$. In the large circle, the arc length is $L = \frac{\theta}{360} \cdot 2\pi R$. We can rewrite that as $L = \frac{\theta}{360} \cdot 2\pi rk$. So $L = k\ell$."

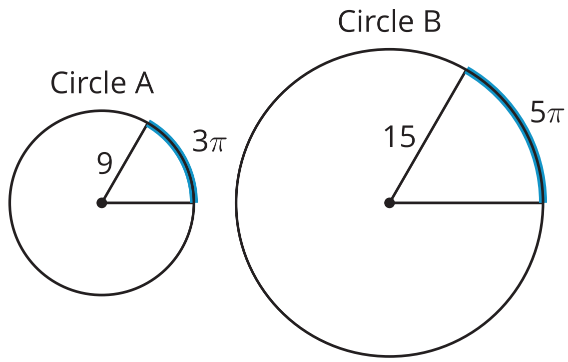
Lin says, "Okay, from here I can show that $\frac{\ell}{r}$ and $\frac{L}{R}$ are equivalent."

1. How does Lin know that the big circle is a dilation of the small circle?
2. How does Lin know that $R = kr$?
3. Why could Diego write $\ell = \frac{\theta}{360} \cdot 2\pi r$?
4. When Diego says that $L = k\ell$, what does that mean in words?
5. Why could Diego say that $L = k\ell$?
6. How can Lin show that $\frac{\ell}{r} = \frac{L}{R}$?

Lesson 14 Summary

If we have the same central angle in two different circles, the length of the arc defined by the angle depends on the size of the circle. So, we can use the relationship between the arc length and the circle's radius to get some information about the size of the central angle.

For example, suppose Circle A has radius 9 units and a central angle that defines an arc with length 3π. Circle B has radius 15 units and a central angle that defines an arc with length 5π. How do the two angles compare?



For the angle in Circle A, the ratio of the arc length to the radius is $\frac{3\pi}{9}$, which can be rewritten as $\frac{\pi}{3}$. For the angle in Circle B, the ratio of the arc length to radius is $\frac{5\pi}{15}$, which can also be written as $\frac{\pi}{3}$. That seems to indicate that the angles are the same size. Let's check.

Circle A's circumference is 18π units. The arc length 3π is $\frac{1}{6}$ of 18π, so the angle measurement must be $\frac{1}{6}$ of 360 degrees, or 60 degrees. Circle B's circumference is 30π units. The arc length 5π is $\frac{1}{6}$ of 30π, so this angle also measures $\frac{1}{6}$ of 360 degrees, or 60 degrees. The two angles are indeed congruent.

