Dealing with Negative Numbers

Let's show that doing the same to each side works for negative numbers too.

8.1 Which Three Go Together: Rational Number Arithmetic

Which three go together? Why do they go together?

A

$$15 = -5 \cdot -3$$

B

$$2 + -5 = -3$$

C

$$4 \cdot 2 - -1 = 9$$

D

$$-3 \cdot -4 = -12$$

8.2 Old and New Ways to Solve

Solve each equation. Be prepared to explain your reasoning.

1. $x + 6 = 4$

2. $x - -4 = -6$

3. $2(x - 1) = -200$

4. $2x + -3 = -23$

8.3 Keeping It True

1. Keep your work secret from your partner. Start with the equation $x = -5$. Do the same thing to each side at least three times to create an equation that has the same solution as the starting equation. Write the equation you ended up with on a slip of paper, and trade equations with your partner.

2. See if you can figure out what steps your partner used to transform $x = -5$ into the equation they gave you. When you think you know, check with your partner to see if you are right.

Lesson 8 Summary

To find a solution to some equations, we can just think about what value in place of the variable would make the equation true. Sometimes we also draw diagrams to reason about the solution. Using balanced hanger diagrams helped us understand that doing the same thing to each side of an equation keeps the equation true. So, another way to solve an equation is to perform the same operation on each side in order to get the variable alone on one side.

Doing the same thing to each side of an equation also works when an equation involves negative numbers. Here are some examples of equations that have negative numbers and steps we could take to solve them.

Example:

$$\begin{aligned} 2(x-5) &= -6 \\ \tfrac{1}{2} \cdot 2(x-5) &= \tfrac{1}{2} \cdot (-6) && \text{Multiply each side by } \tfrac{1}{2} \\ x-5 &= -3 \\ x-5+5 &= -3+5 && \text{Add 5 to each side} \\ x &= 2 \end{aligned}$$

Example:

$$\begin{aligned} -2x+-5 &= 6 \\ -2x+-5--5 &= 6--5 && \text{Subtract -5 from each side} \\ -2x &= 11 \\ -2x \div -2 &= 11 \div -2 && \text{Divide each side by -2} \\ x &= -\tfrac{11}{2} \end{aligned}$$

Doing the same thing to each side maintains equality even if it is not helpful for finding the solution. For example, we could take the equation $-3x + 7 = -8$ and add -2 to each side:

$$\begin{aligned} -3x+7 &= -8 \\ -3x+7+-2 &= -8+-2 && \text{Add -2 to each side} \\ -3x+5 &= -10 \end{aligned}$$

If $-3x + 7 = -8$ is true then $-3x + 5 = -10$ is also true, but we are no closer to a solution than we were before adding -2. We can use moves that maintain equality to make new equations that all have the same solution. Helpful combinations of moves will eventually lead to an equation like $x = 5$, which gives the solution to the original equation (and every equation we wrote in the process of solving).