

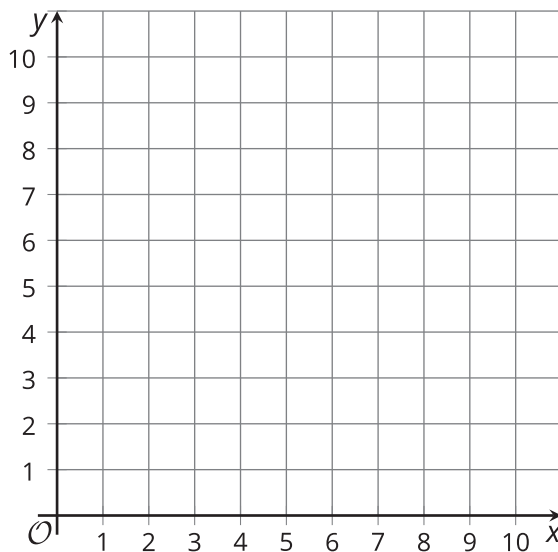


# Weighted Averages

Let's split segments using averages and ratios.

## 15.1 Part Way: Points

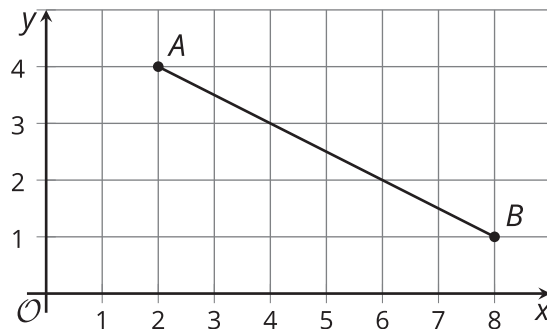
For the questions in this activity, use the coordinate grid if it is helpful to you.



1. What is the midpoint of the segment connecting  $(1, 2)$  and  $(5, 2)$ ?
2. What is the midpoint of the segment connecting  $(5, 2)$  and  $(5, 10)$ ?
3. What is the midpoint of the segment connecting  $(1, 2)$  and  $(5, 10)$ ?

## 15.2 Part Way: Segment

Point  $A$  has coordinates  $(2, 4)$ . Point  $B$  has coordinates  $(8, 1)$ .



1. Find the point that partitions segment  $AB$  in a  $2 : 1$  ratio.
2. Calculate  $C = \frac{1}{3}A + \frac{2}{3}B$ .
3. What do you notice about your answers to the first 2 questions?
4. For 2 new points  $K$  and  $L$ , write an expression for the point that partitions segment  $KL$  in a  $3 : 1$  ratio.

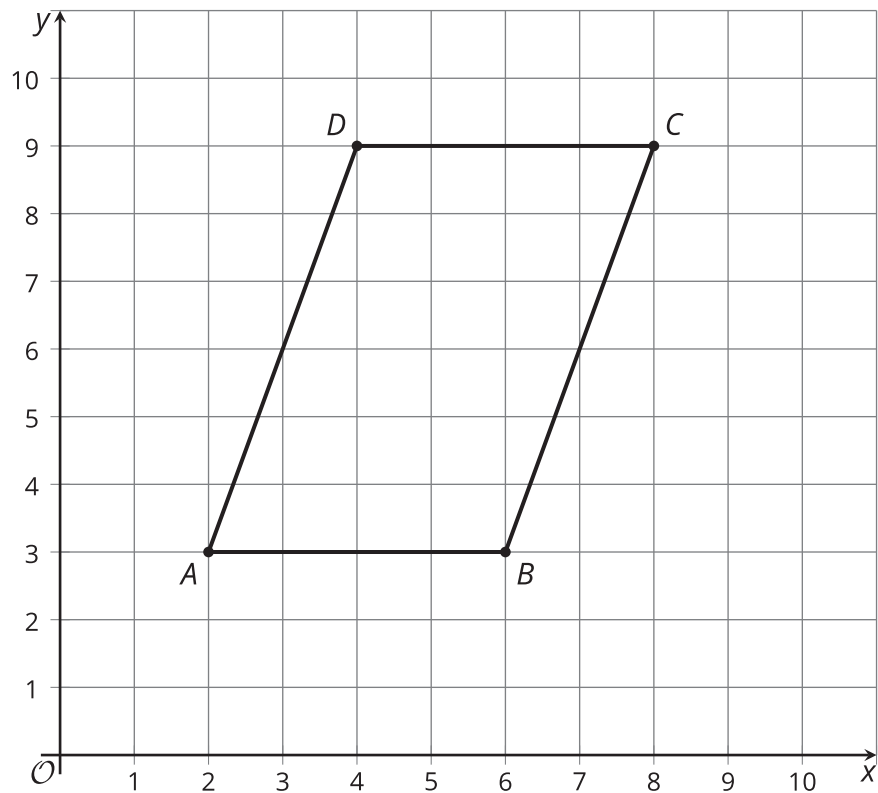
 **Are you ready for more?**

Consider the general quadrilateral  $QRST$  with  $Q(0, 0)$ ,  $R(a, b)$ ,  $S(c, d)$ , and  $T(e, f)$ .

1. Find the midpoints of each side of this quadrilateral.
2. Show that if these midpoints are connected consecutively, the new quadrilateral formed is a parallelogram.

## 15.3 Part Way: Quadrilateral

Here is quadrilateral  $ABCD$ .



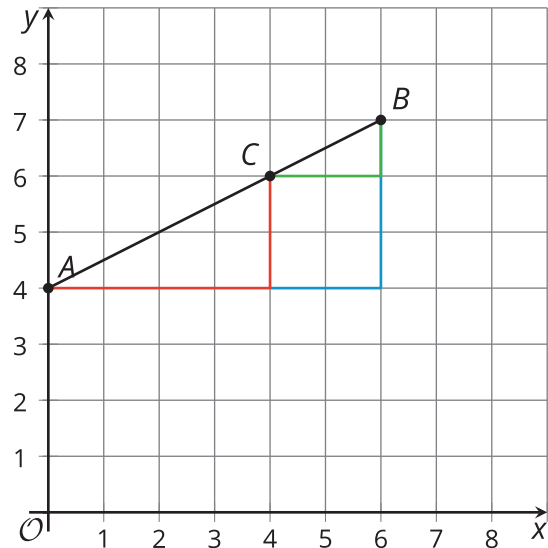
1. Find the point that partitions segment  $AB$  in a 1 : 4 ratio. Label it  $B'$ .
2. Find the point that partitions segment  $AD$  in a 1 : 4 ratio. Label it  $D'$ .
3. Find the point that partitions segment  $AC$  in a 1 : 4 ratio. Label it  $C'$ .
4. Is  $AB'C'D'$  a dilation of  $ABCD$ ? Justify your answer.

## Lesson 15 Summary

To find the midpoint of a line segment, we can average the coordinates of the endpoints. For example, to find the midpoint of the segment from  $A(0, 4)$  to  $B(6, 7)$ , average the coordinates of  $A$  and  $B$ :  $\left(\frac{0+6}{2}, \frac{4+7}{2}\right) = (3, 5.5)$ . Another way to write what we just did is  $\frac{1}{2}(A + B)$  or  $\frac{1}{2}A + \frac{1}{2}B$ .

Now, let's find the point that is  $\frac{2}{3}$  of the way from  $A$  to  $B$ . In other words, we'll find point  $C$  so that segments  $AC$  and  $CB$  are in a 2 : 1 ratio.

In the horizontal direction, segment  $AB$  stretches from  $x = 0$  to  $x = 6$ . The distance from 0 to 6 is 6 units, so we calculate  $\frac{2}{3}$  of 6 to get 4. Point  $C$  will be 4 horizontal units away from  $A$ , which means an  $x$ -coordinate of 4.



In the vertical direction, segment  $AB$  stretches from  $y = 4$  to  $y = 7$ . The distance from 4 to 7 is 3 units, so we can calculate  $\frac{2}{3}$  of 3 to get 2. Point  $C$  must be 2 vertical units away from  $A$ , which means a  $y$ -coordinate of 6.

It is possible to do this all at once by saying  $C = \frac{1}{3}A + \frac{2}{3}B$ . This is called a weighted average. Instead of finding the point in the middle, we want to find a point closer to  $B$  than to  $A$ . So we give point  $B$  more weight—it has a coefficient of  $\frac{2}{3}$  rather than  $\frac{1}{2}$  as in the midpoint calculation. To calculate  $C = \frac{1}{3}A + \frac{2}{3}B$ , substitute and evaluate.

$$\begin{aligned}\frac{1}{3}A + \frac{2}{3}B \\ \frac{1}{3}(0, 4) + \frac{2}{3}(6, 7) \\ \left(0, \frac{4}{3}\right) + \left(4, \frac{14}{3}\right) \\ (4, 6)\end{aligned}$$

Either way, we found that the coordinates of  $C$  are  $(4, 6)$ .