Unit 6, Lesson 20

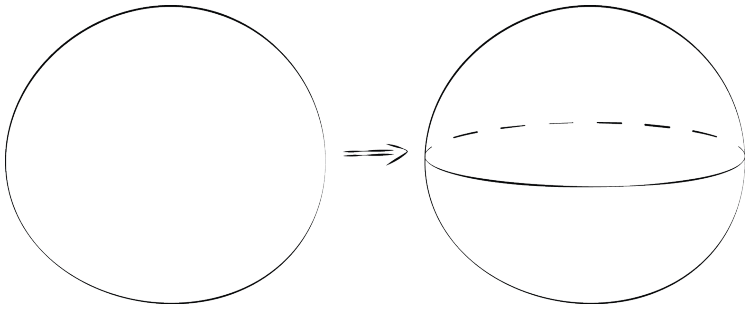
The Volume of a Sphere

Let's explore spheres and their volumes.

20.1 Sketch a Sphere

Here is a method for quickly sketching a sphere:

- Draw a circle.
- Draw an oval in the middle whose edges touch the sphere.

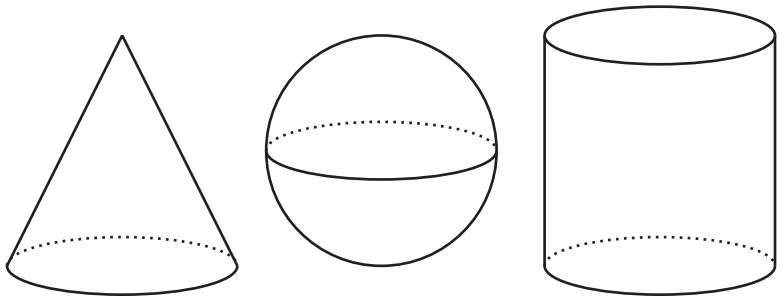


1. Practice sketching some spheres. Sketch a few different sizes.

2. For each sketch, draw a radius and label it r.

20.2 A Sphere in a Cylinder

Here are a cone, a sphere, and a cylinder that all have the same radii and heights.



The radius of the cylinder is 5 units. When necessary, express all answers in terms of π.

1. What is the height of the cylinder?

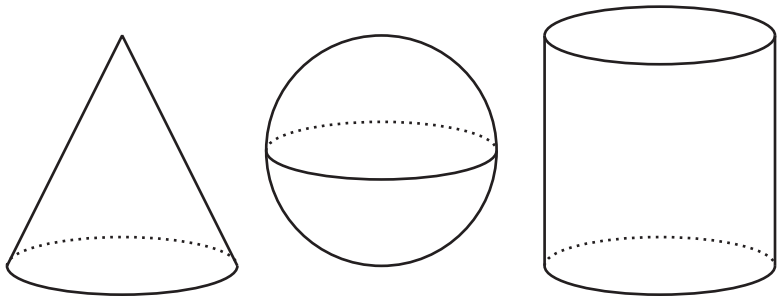
2. What is the volume of the cylinder?

3. What is the volume of the cone?

4. What is the volume of the sphere? Explain your reasoning.

20.3 Spheres in Cylinders

A cone, a sphere, and a cylinder that all have the same radius and height are shown here.



Let the radius of the cylinder be r units. When necessary, express answers in terms of π.

1. What is the height of the cylinder in terms of r?

2. What is the volume of the cylinder in terms of r?

3. What is the volume of the cone in terms of r?

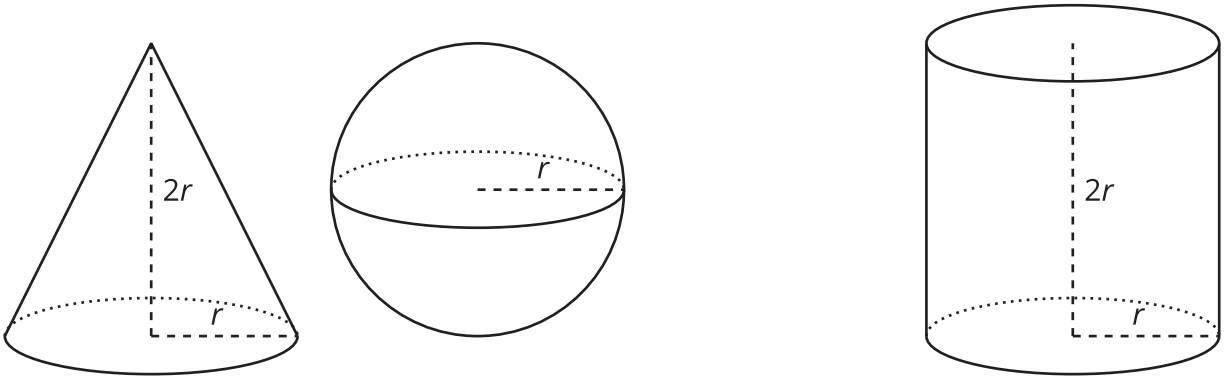
4. What is the volume of the sphere in terms of r?

5. The volume of the cone is $\frac{1}{3}$ the volume of a cylinder. The volume of the sphere is what fraction of the volume of the cylinder?

Lesson 20 Summary

Think about a sphere with radius r units that fits snugly inside a cylinder. The cylinder must then also have a radius of r units and a height of $2r$ units. Using what we have learned about volume, the cylinder has a volume of $\pi r^2 h = \pi r^2 \cdot (2r)$, which is equal to $2\pi r^3$ cubic units.

We know from an earlier lesson that the volume of a cone with the same base and height as a cylinder has $\frac{1}{3}$ of the volume. In this example, such a cone has a volume of $\frac{1}{3} \cdot \pi r^2 \cdot 2r$, or $\frac{2}{3}\pi r^3$ cubic units.



If we filled the cone and sphere with water and then poured that water into the cylinder, the cylinder would be completely filled. That means the volume of the sphere and the volume of the cone add up to the volume of the cylinder. In other words, if V is the volume of the sphere, then

$$V + \frac{2}{3}\pi r^3 = 2\pi r^3$$

This leads to the formula for the volume of the sphere,

$$V = \frac{4}{3}\pi r^3$$