All, Some, or No Solutions

Let’s think about how many solutions an equation can have.

10.1 Notice and Wonder: Equation Solutions

What do you notice? What do you wonder?

$$2t + 5 = 2t + 5$$

$$n + 5 = n + 7$$

10.2 Thinking about Solutions

$$n = n$$

$$2t + 6 = 2(t + 3)$$

$$3(n + 1) = 3n + 1$$

$$\frac{1}{4}(20d + 4) = 5d$$

$$5 - 9 + 3x = -10 + 6 + 3x$$

$$\frac{1}{2} + x = \frac{1}{3} + x$$

$$y \cdot -6 \cdot -3 = 2 \cdot y \cdot 9$$

$$v + 2 = v - 2$$

1. Sort these equations into the two types: true for all values and true for no values.

2. Write the other side of this equation so that this equation is true for all values of u.

$$6(u - 2) + 2 =$$

3. Write the other side of this equation so that this equation is true for no values of u.

$$6(u - 2) + 2 =$$

Are you ready for more?

Consecutive numbers follow one right after the other. An example of three consecutive numbers is 17, 18, and 19. Another example is -100, -99, -98.

How many sets of two or more consecutive positive integers can be added to obtain a sum of 100?

10.3 What's the Equation?

1. Complete each equation so that it is true for all values of x.

   a. $3x + 6 = 3(x +$ ________ $)$

   b. $x - 2 =$ -(________ $- x)$

   c. $\frac{15x - 10}{5} =$ ________ $- 2$

2. Complete each equation so that it is true for no values of x.

   a. $3x + 6 = 3(x +$ ________ $)$

   b. $x - 2 =$ -(________ $- x)$

   c. $\frac{15x - 10}{5} =$ ________ $- 2$

3. Describe how you know whether an equation will be true for all values of x or true for no values of x.

Lesson 10 Summary

An equation is a statement that says that two expressions have an equal value.

The equation $2x = 6$ is a true statement if x is 3. $2 \cdot 3 = 6$

It is a false statement if x is 4: $2 \cdot 4 = 6$

The equation $2x = 6$ has one and only one solution, because there is only one number that you can double to get 6.

Some equations are true no matter what the value of the variable is.

For example, $2x = x + x$ is always true, because if you double a number, that will always be the same as adding the number to itself.

Equations like $2x = x + x$ have an infinite number of solutions. We say that it is true for all values of x.

Some equations have no solutions. For example, $x = x + 1$ has no solutions, because no matter what the value of x is, it can't equal 1 more than itself.

When we solve an equation, we are looking for the values of the variable that make the equation true. When we try to solve the equation, we make valid moves assuming it has a solution. Sometimes we make valid moves and get an equation like this:

$$8 = 7$$

This statement is false, so it must be that the original equation had no solution at all.

