Scope and Sequence for Algebra 1

Students begin the course with one-variable statistics. Data collection and analysis set a tone for the course, of understanding quantities in context, and allows students to access grade-level mathematics with less dependence on prior skills than other topics. Gathering and displaying data, measuring data distribution, and interpreting statistical results encourages students to collaborate, communicate, and explore new tools and routines.

They study linear equations, and systems of linear equations, by modeling relationships in various situations. Students write, evaluate, graph, and solve equations, explaining and validating their reasoning with increased precision. These insights lead into a unit on two-variable statistics in which students examine relationships between variables, using two-way tables, scatter plots, and linear models. From there, they move on to solving and graphing linear inequalities and systems of linear inequalities to represent constraints in situations.

Students deepen their understanding of functions by representing, interpreting, and communicating about them, using function notation, domain and range, average rate of change, and features of graphs. They also see categories of functions, starting with linear functions (including their inverses) and piecewise-defined functions (including absolute-value functions), followed by exponential and quadratic functions. For each function type, students investigate real-world contexts, look closely at the structural attributes of the function, and analyze how these attributes are expressed in different representations.

The course ends with a close look at quadratic equations. Through reasoning, writing equivalent equations, and applying the quadratic formula, students extend their ability to use equations to model relationships and solve problems. Along the way students encounter rational and irrational solutions, deepening their understanding of the real-number system.

Within the classroom activities, students have opportunities to engage in aspects of mathematical modeling. Additionally, modeling prompts are provided for use throughout the course, offering opportunities for students to engage in the full modeling cycle. Implement these in a variety of ways. Please see the *Mathematics Modeling Prompts* section of this Course Guide for a more detailed explanation.

Unit 1: One-Variable Statistics

In this unit, students collect, display, and analyze data using statistics, such as mean, median, interquartile range, and standard deviation.

In grades 6–8, students used histograms, dot plots, and box plots as a way to summarize data and worked with basic measures of center (mean and median) as well as measures of variability (mean absolute deviation and interquartile range). These concepts are revisited in the first two sections of this unit, but with a focus on interpretation and what they reveal about the data in addition to the mechanics of constructing the data displays.

The optional third section is available to familiarize students with spreadsheets and technology that will be used to calculate statistics, such as mean, median, quartiles, and standard deviation, as well as create data displays.

The fourth section introduces additional ways to interpret data using standard deviation and outliers. Students finish the unit by using these tools to compare related data sets using measures of center and measures of variability.

The last lesson gives students a chance to practice their skills by posing a statistical question, designing an experiment, collecting data, and analyzing their data.

Because the first half of the unit mostly revisits material from middle school, a Mid-Unit Assessment is not included in this unit. The *Cool-downs* and *Checkpoints* can be used to monitor student understanding.

In this unit, only the population standard deviation is used. Sample standard deviation is introduced in a later course.

GeoGebra's spreadsheets are chosen for their versatility for the on-level mathematics in this course. While other spreadsheet programs have additional functionality and uses, they are limited in other ways. That said, please adapt the materials to the needs of your students.

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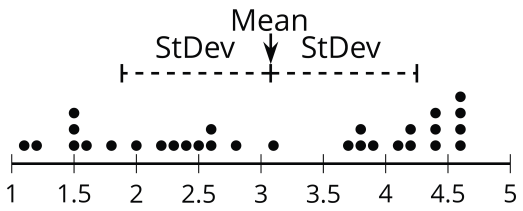


Section A: Getting to Know You

- Lesson 1: Getting to Know You
- Lesson 2: Data Representations
- Lesson 3: A Gallery of Data

Section B: Distribution Shapes

- Lesson 4: The Shape of Distributions
- Lesson 5: Calculating Measures of Center and Variability

Section C: How to Use Spreadsheets

- Lesson 6: Mystery Computations
- Lesson 7: Spreadsheet Computations
- Lesson 8: Spreadsheet Shortcuts

Section D: Manipulating Data

- Lesson 9: Technological Graphing
- Lesson 10: The Effect of Extremes
- Lesson 11: Comparing and Contrasting Data Distributions
- Lesson 12: Standard Deviation
- Lesson 13: More Standard Deviation
- Lesson 14: Outliers
- Lesson 15: Comparing Data Sets

Section E: Let's Put It to Work

- Lesson 16: Analyzing Data

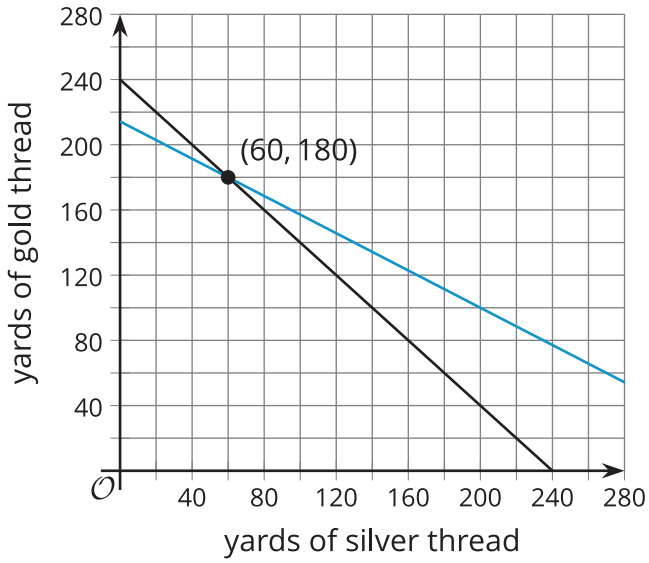
Unit 2: Linear Equations and Systems

In this unit, students examine solving and graphing linear equations and systems of linear equations.

The unit builds on learning from middle school when students used variables to write equations, manipulated equations using valid moves such as the distributive property, and solved basic systems of linear equations using graphs and substitution.

In the first section, students recall writing equations to represent situations. In the second section, they use valid moves to write equivalent equations that can be used to solve for unknown values or to isolate variables. The third section examines solving systems of equations using graphs, substitution for variables, and elimination of variables. Students use their understanding of writing equivalent equations to understand why each of the methods works for finding the

solution.



Section A: Writing and Modeling with Equations

- Lesson 1: Planning a Party
- Lesson 2: Writing Equations to Model Relationships (Part 1)
- Lesson 3: Writing Equations to Model Relationships (Part 2)
- Lesson 4: Equations and Their Solutions
- Lesson 5: Equations and Their Graphs

Section B: Manipulating Equations and Understanding Their Structure

- Lesson 6: Equivalent Equations
- Lesson 7: Explaining Steps for Rewriting Equations
- Lesson 8: Which Variable to Solve for? (Part 1)
- Lesson 9: Which Variable to Solve for? (Part 2)
- Lesson 10: Connecting Equations to Graphs (Part 1)
- Lesson 11: Connecting Equations to Graphs (Part 2)

Section C: Systems of Linear Equations in Two Variables

- Lesson 12: Writing and Graphing Systems of Linear Equations
- Lesson 13: Solving Systems by Substitution
- Lesson 14: Solving Systems by Elimination (Part 1)
- Lesson 15: Solving Systems by Elimination (Part 2)
- Lesson 16: Solving Systems by Elimination (Part 3)
- Lesson 17: Systems of Linear Equations and Their Solutions

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Section D: Let's Put It to Work

- Lesson 18: Asking about Solving Systems
- Lesson 19: Linear Patterns

Unit 3: Two-Variable Statistics

In this unit, students use statistical methods to look for associations in bivariate data. The unit begins with students analyzing categorical data arranged in two-way tables. Students use the relative frequencies of the combinations of those categorical variables to check for evidence of any associations in the data.

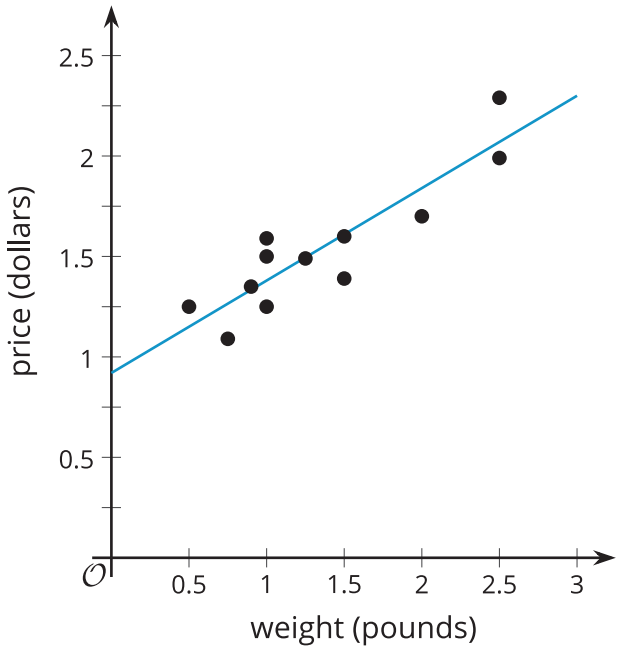
The unit then transitions to bivariate numerical data, which are visualized using scatter plots and lines of best fit. Students use technology to compute the lines of best fit and observe how well the linear models match the data. Residuals and correlation coefficients are used to quantify the goodness of fit for linear models.

The unit closes with an exploration of the difference between correlation and causal relationships, and it is also an opportunity to apply this learning to areas of interest, like anthropology and sports.

In grade 8, students informally constructed scatter plots and lines of fit, noticed linear patterns, and observed associations in categorical data using two-way tables. In this unit, students build on this previous knowledge by assessing how well a linear model matches the data by using residuals as well as the correlation coefficient for best-fit lines (found using technology).

There are opportunities to practice concepts from a previous unit by interpreting the slope and intercept of a linear model in context as well as using the models to predict one variable given information about the other.

$y = 0.46x + 0.92$



Section A: Two-Way Tables

- Lesson 1: Two-Way Tables
- Lesson 2: Relative Frequency Tables
- Lesson 3: Associations in Categorical Data

Section B: Scatter Plots

- Lesson 4: Linear Models
- Lesson 5: Fitting Lines
- Lesson 6: Residuals

Section C: Correlation Coefficients

- Lesson 7: The Correlation Coefficient
- Lesson 8: Using the Correlation Coefficient
- Lesson 9: Causal Relationships

Section D: Let's Put It to Work

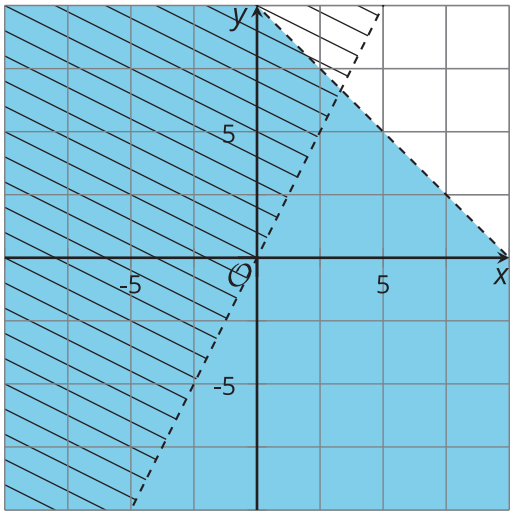
- Lesson 10: Fossils and Flags

Unit 4: Linear Inequalities and Systems

In this unit, students examine solving and graphing linear inequalities and systems of linear inequalities.

The unit builds on concepts from middle school when students write and solve inequalities by reasoning about quantities. It further builds on concepts from an earlier unit in which students solve linear equations and systems of equations by writing equivalent equations.

To start, students solve linear inequalities in one variable and graph the solutions on a number line by writing equivalent inequalities. In the second section, they solve linear inequalities with two variables by looking at the related equation, graphing it on a coordinate plane, and testing points on either side of the line to determine the solution region. The third section is about solving systems of linear inequalities considering multiple linear inequalities as conditions for situations and finding a solution region that satisfies all of the inequalities.



Section A: Linear Inequalities in One Variable

- Lesson 1: Representing Situations with Inequalities
- Lesson 2: Solutions to Inequalities in One Variable
- Lesson 3: Writing and Solving Inequalities in One Variable

Section B: Linear Inequalities in Two Variables

- Lesson 4: Graphing Linear Inequalities in Two Variables (Part 1)
- Lesson 5: Graphing Linear Inequalities in Two Variables (Part 2)
- Lesson 6: Solving Problems with Inequalities in Two Variables

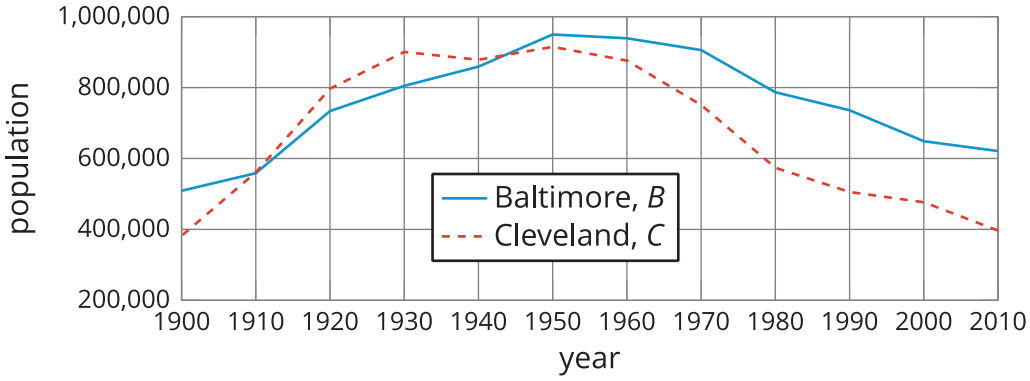
Section C: Systems of Linear Inequalities in Two Variables

- Lesson 7: Solutions to Systems of Linear Inequalities in Two Variables
- Lesson 8: Solving Problems with Systems of Linear Inequalities in Two Variables
- Lesson 9: Modeling with Systems of Inequalities in Two Variables

Unit 5: Functions

In this unit, students expand and deepen their understanding of functions. They begin with a reminder of the definition of a function (a rule that assigns exactly one output to each input) that they previously saw in grade 8, then get familiar with function notation and use it to compare and analyze functions, write rules for functions, and solve for inputs or outputs.

Then, students explore graphs of functions to describe features such as “maximum,” “minimum,” “intercepts,” “increasing,” “decreasing,” and “average rate of change” and make connections between the graphs and real-life situations. They use situations to discuss the domain and range of a function and make sense of piecewise-defined functions. In particular, students examine the absolute value function and some basic transformations of it. Later, students explore inverses of linear functions as a way to find corresponding input values when output values are known. Throughout the unit, students have chances to mathematically model real-world situations.



Section A: Functions and Their Representations

- Lesson 1: Describing and Graphing Situations
- Lesson 2: Function Notation
- Lesson 3: Interpreting and Using Function Notation
- Lesson 4: Using Function Notation to Describe Rules (Part 1)
- Lesson 5: Using Function Notation to Describe Rules (Part 2)

Section B: Analyzing and Creating Graphs of Functions

- Lesson 6: Features of Graphs
- Lesson 7: Using Graphs to Find Average Rate of Change

- Lesson 8: Interpreting and Creating Graphs
- Lesson 9: Comparing Graphs

Section C: A Closer Look at Inputs and Outputs

- Lesson 10: Domain and Range (Part 1)
- Lesson 11: Domain and Range (Part 2)
- Lesson 12: Piecewise Functions
- Lesson 13: Absolute Value Functions (Part 1)
- Lesson 14: Absolute Value Functions (Part 2)

Section D: Inverse Functions

- Lesson 15: Inverse Functions
- Lesson 16: Finding and Interpreting Inverse Functions
- Lesson 17: Writing Inverse Functions to Solve Problems

Section E: Let's Put It to Work

- Lesson 18: Using Functions to Model Battery Power

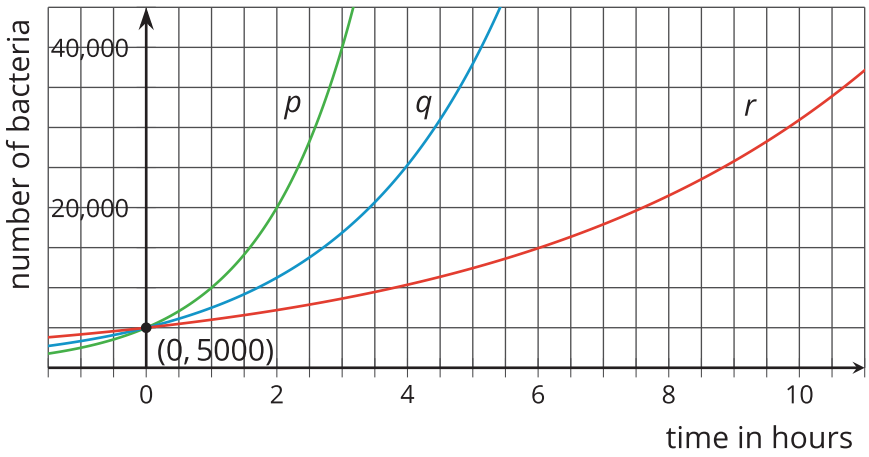
Unit 6: Introduction to Exponential Functions

In this unit, students build on their understanding of linear functions, properties of exponents, and percent change to explore exponential relationships. Students learn that exponential relationships are characterized by a constant quotient over equal intervals, and compare it to linear relationships, which are characterized by a constant difference over equal intervals. They encounter contexts that change exponentially. These contexts are presented verbally and with tables and graphs. They construct equations and use them to model situations and solve problems. At first, students investigate these exponential relationships without using function notation and language so that they can focus on gaining an appreciation for critical properties and characteristics of exponential relationships.

Later, students view these relationships as functions and employ the notation and terminology of functions. They study graphs of exponential functions both in terms of contexts that they represent and abstract functions that don't represent a particular context, observing the effect of different values of a and b on the graph of the function f represented by $f(x) = ab^x$.

The contexts used early in this unit lead to functions where the domain is the integers. In some situations, students will interpret exponential expressions with fractional values in the exponent using graphing, but the connection to roots or logarithms is left for a later course.

Note on materials: Throughout this unit, students should have access to a calculator with an exponent button. Access to graphing technology is necessary for some activities and encouraged throughout the unit. Examples of graphing technology include a handheld graphing calculator, a computer with a graphing calculator application installed, or an internet-enabled device with access to a site like desmos.com/calculator or geogebra.org/graphing. Interactive applets are embedded throughout, and a graphing calculator tool is accessible in the Math Tools in the digital version.



Section A: Looking at Growth

- Lesson 1: Growing and Growing
- Lesson 2: Patterns of Growth

Section B: A New Kind of Relationship

- Lesson 3: Representing Exponential Growth
- Lesson 4: Representing Exponential Decay
- Lesson 5: Understanding Decay
- Lesson 6: Analyzing Graphs
- Lesson 7: Using Negative Exponents

Section C: Exponential Functions

- Lesson 8: Exponential Situations as Functions
- Lesson 9: Interpreting Exponential Functions
- Lesson 10: Looking at Rates of Change
- Lesson 11: Modeling Exponential Behavior
- Lesson 12: Reasoning about Exponential Graphs (Part 1)
- Lesson 13: Reasoning about Exponential Graphs (Part 2)

Section D: Percent Growth and Decay

- Lesson 14: Recalling Percent Change
- Lesson 15: Functions Involving Percent Change
- Lesson 16: Compounding Interest
- Lesson 17: Different Compounding Intervals
- Lesson 18: Expressed in Different Ways

Section E: Comparing Linear and Exponential Functions

- Lesson 19: Which One Changes Faster?

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- Lesson 20: Changes over Equal Intervals

Section F: Let's Put It to Work

- Lesson 21: Predicting Populations

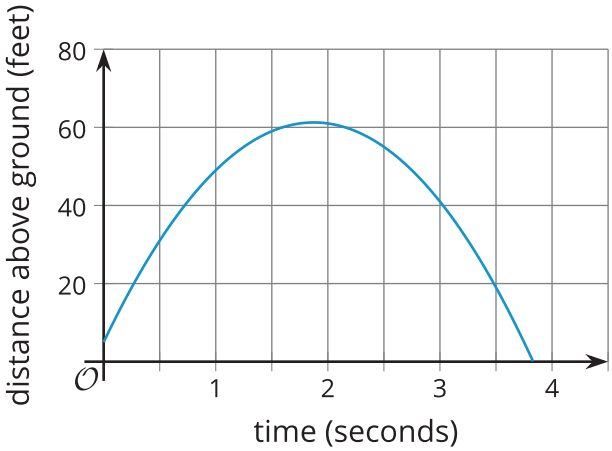
Unit 7: Introduction to Quadratic Functions

Prior to this unit, students have studied what it means for a relationship to be a function, used function notation, and investigated linear and exponential functions. In this unit, they look at some patterns that grow quadratically and contrast this growth with linear and exponential growth. They further observe that eventually these quadratic patterns grow more quickly than do linear patterns but more slowly than exponential patterns grow.

Students examine the important example of free-falling objects whose height over time can be modeled with quadratic functions. They use tables, graphs, and equations to describe the movement of these objects, eventually looking at the situation in which a projectile is launched upward. They interpret the meaning of each term in this context and work toward understanding how the coefficients influence the shape of the graph. Additional situations, such as revenue and area, are also introduced.

Next, students examine standard, factored, and vertex forms of quadratic functions. They recognize what information about the graph is easily obtained from each form and how the different values in each form influence the graph. In particular, they begin to generalize ideas of how horizontal and vertical translation, as well as vertical and horizontal stretching of graphs, relate to modifying the equation of a function.

Note on materials: Access to graphing technology is necessary for many activities. Examples of graphing technology are: a handheld graphing calculator, a computer with a graphing calculator application installed, and an internet-enabled device with access to a site like desmos.com/calculator or geogebra.org/graphing. For students using the digital version of these materials, a separate graphing calculator tool isn't necessary. Interactive applets are embedded throughout, and a graphing calculator tool is accessible in the student math tools.



Section A: A Different Kind of Change

- Lesson 1: A Different Kind of Change
- Lesson 2: How Does It Change?

Section B: Quadratic Functions

- Lesson 3: Building Quadratic Functions from Geometric Patterns
- Lesson 4: Comparing Quadratic and Exponential Functions

- Lesson 5: Building Quadratic Functions to Describe Situations (Part 1)
- Lesson 6: Building Quadratic Functions to Describe Situations (Part 2)
- Lesson 7: Building Quadratic Functions to Describe Situations (Part 3)

Section C: Working with Quadratic Expressions

- Lesson 8: Equivalent Quadratic Expressions
- Lesson 9: Standard Form and Factored Form
- Lesson 10: Graphs of Functions in Standard and Factored Forms

Section D: Features of Graphs of Quadratic Functions

- Lesson 11: Graphing from the Factored Form
- Lesson 12: Graphing the Standard Form (Part 1)
- Lesson 13: Graphing the Standard Form (Part 2)
- Lesson 14: Graphs That Represent Situations
- Lesson 15: Vertex Form
- Lesson 16: Graphing from the Vertex Form
- Lesson 17: Changing the Vertex

Unit 8: Quadratic Equations

In this unit, students interpret, write, and solve equations algebraically.

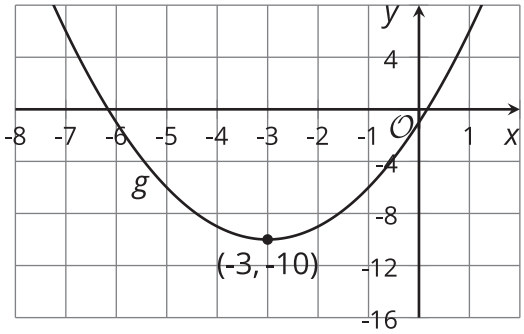
Previously, students represented quadratic functions using expressions, tables, and descriptions. They connected important features of graphs to standard, factored, and vertex forms and expanded expressions into standard form.

Students begin with solving quadratic equations through reasoning without much algebraic manipulation. Then, they examine solving using the zero product property for quadratic equations that can be written in factored form. They notice patterns that help them rewrite quadratic expressions in factored form and recognize that not all of them are easily factorable.

This motivates finding another process of solving quadratic equations. Students recognize the benefits of equations of the form $(x - n)^2 = q$ and develop the method of completing the square. Then, they generalize this process to the quadratic formula.

Throughout the unit, students analyze quadratic equations to recognize that there can be 0, 1, or 2 solutions. Solutions can be rational, irrational, or combinations of these. Students interpret the solutions that arise in different contexts.

The unit concludes with rewriting quadratic expressions from standard form into vertex form to find maximum and minimum values, then apply all of their understanding to applied problems.



Section A: Finding Unknown Inputs

- Lesson 1: Finding Unknown Inputs
- Lesson 2: When and Why Do We Write Quadratic Equations?

Section B: Solving Quadratic Equations

- Lesson 3: Solving Quadratic Equations by Reasoning
- Lesson 4: Solving Quadratic Equations with the Zero Product Property
- Lesson 5: How Many Solutions?
- Lesson 6: Rewriting Quadratic Expressions in Factored Form (Part 1)
- Lesson 7: Rewriting Quadratic Expressions in Factored Form (Part 2)
- Lesson 8: Rewriting Quadratic Expressions in Factored Form (Part 3)
- Lesson 9: Solving Quadratic Equations by Using Factored Form
- Lesson 10: Rewriting Quadratic Expressions in Factored Form (Part 4)

Section C: Completing the Square

- Lesson 11: What Are Perfect Squares?
- Lesson 12: Completing the Square (Part 1)
- Lesson 13: Completing the Square (Part 2)
- Lesson 14: Completing the Square (Part 3)
- Lesson 15: Quadratic Equations with Irrational Solutions

Section D: The Quadratic Formula

- Lesson 16: The Quadratic Formula
- Lesson 17: Applying the Quadratic Formula (Part 1)
- Lesson 18: Applying the Quadratic Formula (Part 2)
- Lesson 19: Deriving the Quadratic Formula
- Lesson 20: Rational and Irrational Solutions
- Lesson 21: Sums and Products of Rational and Irrational Numbers

Section E: Vertex Form Revisited

- Lesson 22: Rewriting Quadratic Expressions in Vertex Form

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- Lesson 23: Using Quadratic Expressions in Vertex Form to Solve Problems

Section F: Let's Put It to Work

- Lesson 24: Using Quadratic Equations to Model Situations and Solve Problems

Pacing Guide

Number of days includes assessments. Upper bound of range includes optional lessons.
Time for modeling prompts is not included.

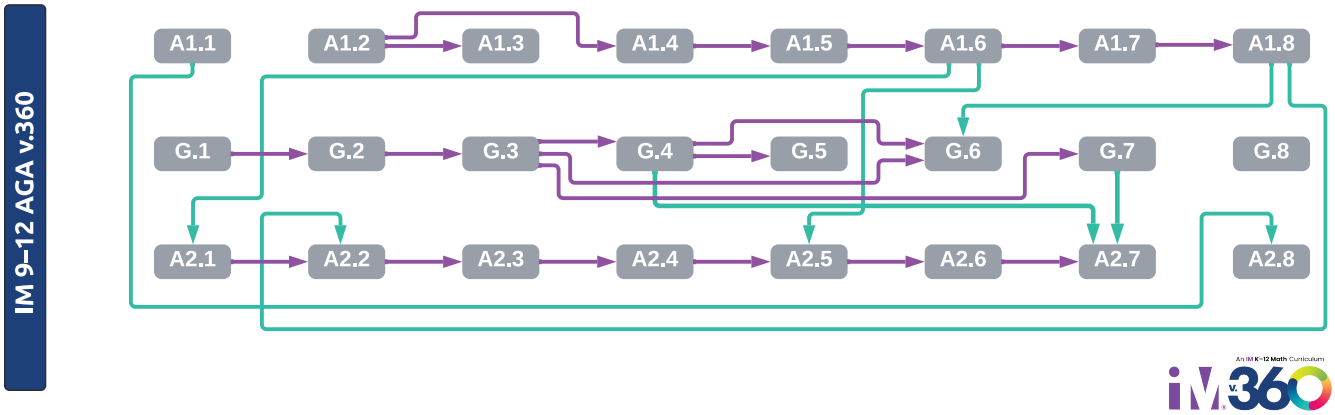
	Algebra 1	Geometry	Algebra 2
week 1	Unit 1 One-variable Statistics 13–18 days Optional Lessons: 2, 5, 6, 7, 8	Unit 1 (MA) Constructions and Rigid Transformations 22–25 days Optional Lessons: 8, 18, 22	Unit 1 Sequences and Functions 12–13 days Optional Lesson: 4
week 2			
week 3			
week 4	Unit 2 Linear Equations and Systems 16–21 days Optional Lessons: 2, 4, 5, 18, 19		Unit 2 Polynomials 17 days Optional Lessons: none
week 5			
week 6		Unit 2 Congruence 16–17 days Optional Lesson: 11	
week 7			Unit 3 Rational Functions & Identities 13 days Optional Lessons: none
week 8	Unit 3 Two-variable Statistics 11–12 days Optional Lesson: 10		
week 9		Unit 3 Similarity 16–19 days Optional Lessons: 2, 10, 12	
week 10			Unit 4 Complex Nums & Rat Exponents 15–22 days Optional Lessons: 1, 2, 9, 14, 16, 19, 20
week 11	Unit 4 Linear Inequalities and Systems 11 days Optional Lessons: none		
week 12			
week 13		Unit 4 Right Triangle Trigonometry 12–14 days Optional Lessons: 2, 3	
week 14	Unit 5 (MA) Functions 21 days Optional Lessons: none		Unit 5 (MA) Exponential Functions and Equations 18–21 days Optional Lessons: 2, 7, 18
week 15			
week 16		Unit 5 Solid Geometry 20 days Optional Lessons: none	
week 17			
week 18	Unit 6 (MA) Introduction to Exponential Functions 22–24 days Optional Lesson: 13, 14		Unit 6 Transformations of Functions 16–17 days Optional Lessons: 14
week 19			
week 20		Unit 6 Coordinate Geometry 20 days Optional Lessons: none	
week 21			Unit 7 (MA) Trigonometric Functions 22–23 days Optional Lessons: 13
week 22			
week 23	Unit 7 (MA) Introduction to Quadratic Functions 17–20 days Optional Lesson: 13, 14, 16		
week 24		Unit 7 Circles 16 days Optional Lessons: none	
week 25			
week 26			Unit 8 (MA) Statistical Inferences 17–18 days Optional Lesson: 4
week 27	Unit 8 (MA) Quadratic Equations 26–27 days Optional Lessons: 18	Unit 8 Conditional Probability 11–13 days Optional Lessons: 1, 11	
week 28			
week 29			
week 30			
week 31			

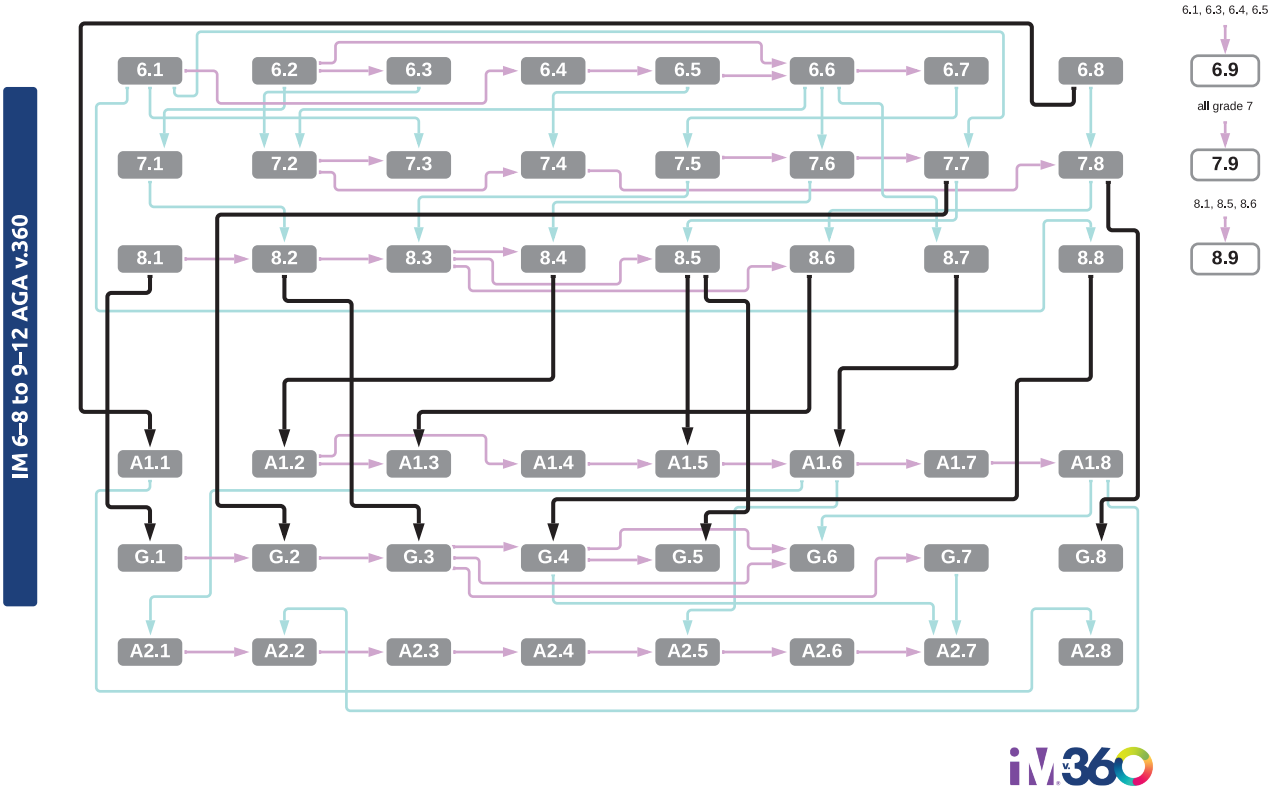
(MA) = Unit has Mid-Unit Assessment

Total number of days = Lessons + Assessments – Optional Lessons

Algebra 1 = 137, Geometry = 133, Algebra 2 = 130

Dependency Chart



In the unit dependency chart, an arrow indicates that a particular unit is designed for students who already know the material in a previous unit. Reversing the order of the units would have a negative effect on mathematical or pedagogical coherence. For example, there is an arrow from A1.6 to A1.7, because when quadratic functions are introduced, they are contrasted with exponential functions, assuming that students are already familiar with exponential functions.

The following chart shows unit dependencies between 6–8 and Algebra 1.