Functions Involving Percent Change

Let's investigate what happens when we repeatedly apply a percent increase to a quantity.

15.1 Dandy Discounts

All books at a bookstore are 25% off. Priya bought a book originally priced at $32. The cashier applied the storewide discount and then took another 25% off for a coupon that Priya brought. If there was no sales tax, how much did Priya pay for the book? Show your reasoning.

15.2 Owing Interests

To get a new computer, a recent college graduate obtains a loan of $450. She agrees to pay 18% annual interest, which will apply to any money she owes. She makes no payments during the first year.

1. How much will she owe at the end of one year? Show your reasoning.

2. Assuming she continues to make no payments to the lender, how much will she owe at the end of two years? Three years?

3. To find the amount owed at the end of the third year, a student started by writing:

$$[\text{Year 3 Amount}] = [\text{Year 2 Amount}] + [\text{Year 2 Amount}] \cdot (0.18)$$

and ended with

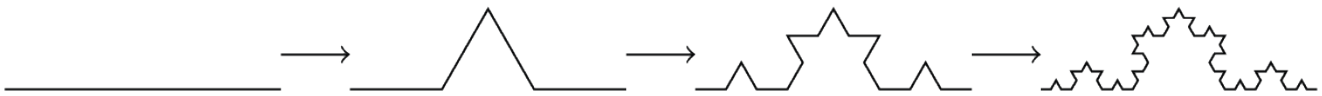
$$[\text{Year 3 Amount}] = 450 \cdot (1.18) \cdot (1.18) \cdot (1.18)$$

Does her final expression correctly reflect the amount owed at the end of the third year? Explain or show your reasoning.

4. Write an expression for the amount she owes at the end of x years without payment.

Are you ready for more?

Start with a line segment of length 1 unit. Make a new shape by taking the middle third of the line segment and replacing it by two line segments of the same length to reconnect the two pieces. Repeat this process over and over, replacing the middle third of each of the remaining line segments with two segments each of the same length as the segment they replaced, as shown in the figure.



What is the total length of the figure after one iteration of this process (the second shape in the diagram)? After two iterations? After n iterations? Experiment with the value of your expression for large values of n.

15.3 Comparing Loans

Suppose three people each have taken loans of $1,000, but they each pay different annual interest rates.

1. For each loan, write an expression, using only multiplication, for the amount owed at the end of each year if no payments are made.

years without payment	Loan A 12%	Loan B 24%	Loan C 30.6%
1			
2			
3			
10			
x			

2. Use graphing technology to plot the graphs of the account balances.
3. Based on your graph, about how many years would it take for the original unpaid balance of each loan to double?

15.4 Comparing Average Rates of Change

The functions a, b, and c represent the amount owed (in dollars) for Loans A, B, and C, respectively. The input for the functions is t, the number of years without payments.

1. For each loan, find the average rate of change per year between:
   a. the start of the loan and the end of the second year.

   b. the end of the tenth year and the end of the twelfth year.

2. How do the average rates of change for the three loans compare in each of the two-year intervals?

Lesson 15 Summary

When we borrow money from a lender, the lender usually charges *interest*, a percentage of the borrowed amount as payment for allowing us to use the money. The interest is usually calculated at a regular interval of time (for example, daily, monthly, or yearly).

Suppose you received a loan of \$500 and the interest rate is 15%, calculated at the end of each year. If you make no other purchases or payments, the amount owed after one year would be $500 + (0.15) \cdot 500$, or $500 \cdot (1 + 0.15)$. If you continue to make no payments or other purchases in the second year, the amount owed would increase by another 15%. The table shows the calculation of the amount owed for the first three years.

time in years	amount owed in dollars
1	$500 \cdot (1 + 0.15)$
2	$500 \cdot (1 + 0.15)(1 + 0.15)$, or $500 \cdot (1 + 0.15)^2$
3	$500 \cdot (1 + 0.15)(1 + 0.15)(1 + 0.15)$, or $500 \cdot (1 + 0.15)^3$

The pattern here continues. Each additional year means multiplication by another factor of $(1 + 0.15)$. With no further purchases or payments, after t years the debt in dollars is given by the expression:

$$500 \cdot (1 + 0.15)^t$$

In this representation, we might leave the growth factor as $(1 + 0.15)$ rather than combining it to 1.15 so that the percentage increase is easier to see. In other situations, it may make sense to write it as 1.15, depending on what is being emphasized. Because exponential functions eventually grow very quickly, leaving a debt unpaid can be very costly.