Unit 2, Lesson 7

Angle-Side-Angle Triangle Congruence

Let's see if we can prove other sets of measurements that guarantee triangles are congruent, and apply those theorems.

7.1 Notice and Wonder: Assertion

Assertion: Through 2 distinct points passes a unique line. Two lines are said to be *distinct* if there is at least 1 point that belongs to one but not the other. Otherwise, we say the lines are the same. Lines that have no point in common are said to be *parallel*.

Conclusion: Given two distinct lines, either they are parallel, or they have exactly 1 point in common.

What do you notice? What do you wonder?

7.2 Proving the Angle-Side-Angle Triangle Congruence Theorem

1. Two triangles have 2 pairs of corresponding angles congruent, and the corresponding sides between those angles are congruent. Sketch two triangles that fit this description.

2. Label the triangles WXY and DEF so that angle W is congruent to angle D, angle X is congruent to angle E, and side WX is congruent to side DE.
3. Use a sequence of rigid motions to take triangle WXY onto triangle DEF. For each step, explain how you know that one or more vertices will line up.

7.3 What Do We Know for Sure about Parallelograms?

Quadrilateral $ABCD$ is a parallelogram. By definition, that means that segment AB is parallel to segment CD, and segment BC is parallel to segment AD.

1. Sketch parallelogram $ABCD$ and then draw an auxiliary line to show how $ABCD$ can be decomposed into 2 triangles.

2. Prove that the 2 triangles you created are congruent.

3. Write a 1–2 sentence summary of why one pair of opposite sides of a parallelogram must be congruent.

Are you ready for more?

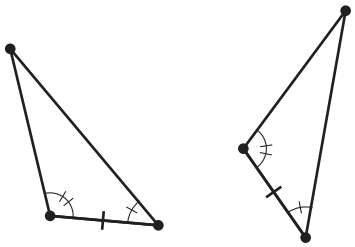
When we have 3 consecutive vertices of a polygon A, B, and C so that the triangle ABC lies entirely inside the polygon, we call B an *ear* of the polygon.

1. How many ears does a parallelogram have?
2. Draw a quadrilateral that has fewer ears than a parallelogram.
3. In 1975, American mathematician Gary Meisters proved that every polygon has at least 2 ears. Draw a hexagon with only 2 ears.

Lesson 7 Summary

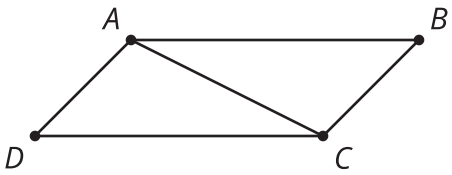
We know that in two triangles, if 2 pairs of corresponding sides and the pair of corresponding angles between the sides are congruent, then the triangles must be congruent. But we don't always know that 2 pairs of corresponding sides are congruent. For example, when proving that opposite sides are congruent in any parallelogram, we only have information about 1 pair of corresponding sides. That is why we need ways other than the Side-Angle-Side Triangle Congruence Theorem to prove triangles are congruent.

In two triangles, if 2 pairs of corresponding angles and the pair of corresponding sides between the angles are congruent, then the triangles must be congruent. This is called the *Angle-Side-Angle Triangle Congruence Theorem*.



When proving that two triangles are congruent, look at the diagram and given information, and think about whether it will be easier to find 2 pairs of corresponding angles that are congruent or 2 pairs of corresponding sides that are congruent. Then check if there is enough information to use the Angle-Side-Angle Triangle Congruence Theorem or the Side-Angle-Side Triangle Congruence Theorem.

The Angle-Side-Angle Triangle Congruence Theorem can be used to prove that, in a parallelogram, opposite sides are congruent. A parallelogram is defined to be a quadrilateral in which the 2 pairs of opposite sides are parallel.



We could prove that triangles ABC and CDA are congruent by the Angle-Side-Angle Triangle Congruence Theorem. Then we can say segment AD is congruent to segment CB because they are corresponding parts of congruent triangles.