

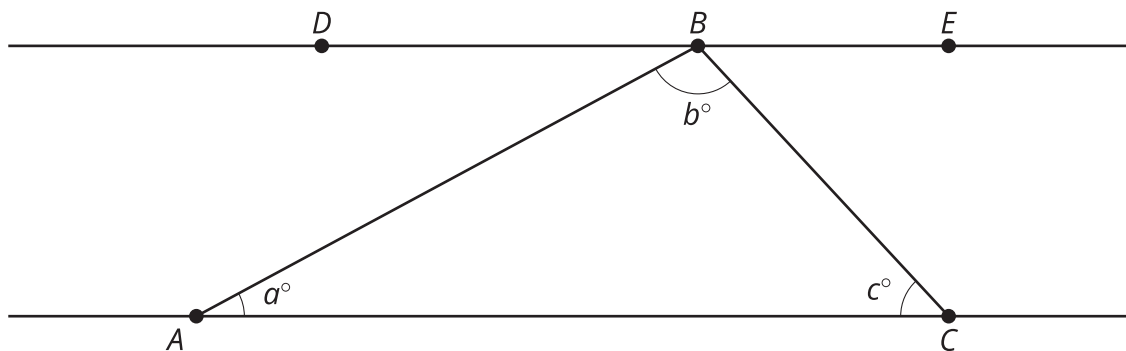


# Parallel Lines and the Angles in a Triangle

Let's see why the angles in a triangle add to 180 degrees.

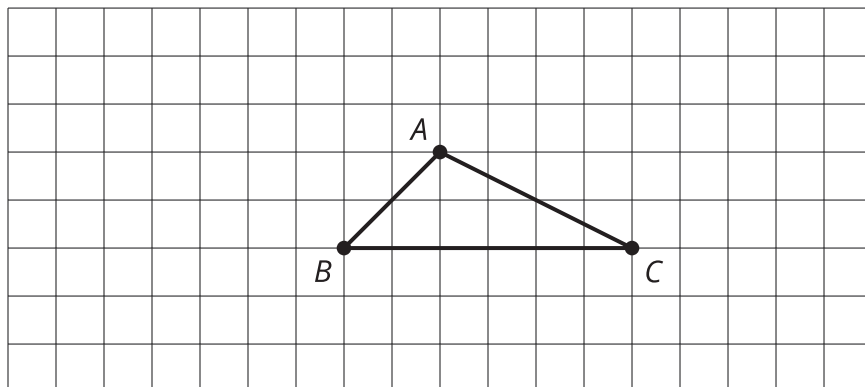
## 16.1 All the Angles

Here is triangle  $ABC$ . Line  $DE$  is parallel to line  $AC$ .



## 16.2 Angle Plus Two

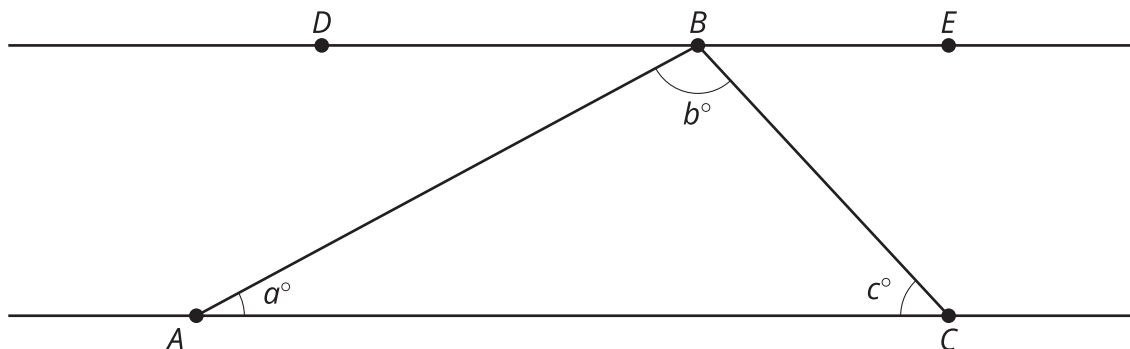
Here is triangle  $ABC$ .



1. Rotate triangle  $ABC$   $180^\circ$  around the midpoint of side  $AC$ . Label the new vertex  $D$ .
2. Rotate triangle  $ABC$   $180^\circ$  around the midpoint of side  $AB$ . Label the new vertex  $E$ .
3. Look at angles  $EAB$ ,  $BAC$ , and  $CAD$ . Without measuring, write what you think is the sum of the measures of these angles. Explain or show your reasoning.
4. Is the measure of angle  $EAB$  equal to the measure of any angle in triangle  $ABC$ ? If so, which one? Explain your reasoning.
5. Is the measure of angle  $CAD$  equal to the measure of any angle in triangle  $ABC$ ? If so, which one? Explain your reasoning.
6. What is the sum of the measures of angles  $ABC$ ,  $BAC$ , and  $ACB$ ? Explain your reasoning.

## 16.3 Every Triangle in the World

Here is triangle  $ABC$ . Line  $DE$  is parallel to line  $AC$ .



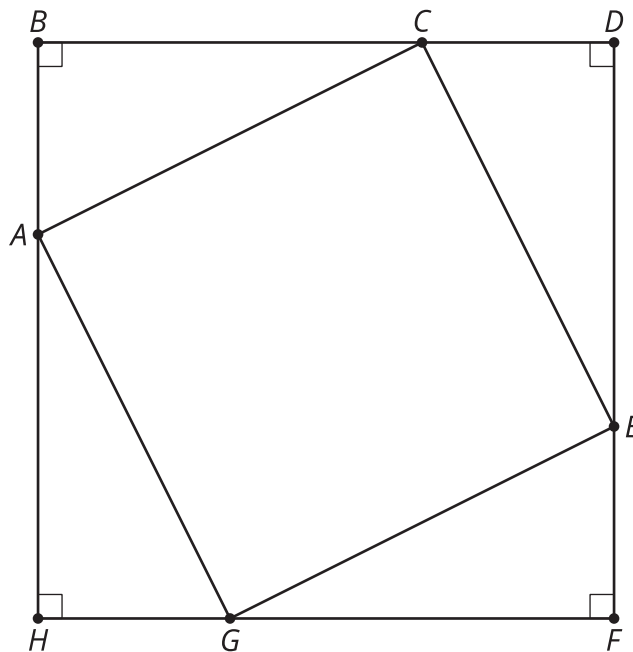
1. What is the sum of the measures of angle  $DBA$ , angle  $ABC$ , and angle  $CBE$ ?
2. Use your answer to explain why  $a + b + c = 180$ .
3. Explain why your argument will work for any triangle: that is, explain why the sum of the angle measures in any triangle is  $180^\circ$ .

 **Are you ready for more?**

1. Using a ruler, create a few quadrilaterals. Use a protractor to measure the four angles inside the quadrilateral. What is the sum of these four angle measures?
2. Come up with an explanation for why anything you notice must be true. (Hint: draw one diagonal in each quadrilateral.)

## 16.4 Four Triangles Revisited

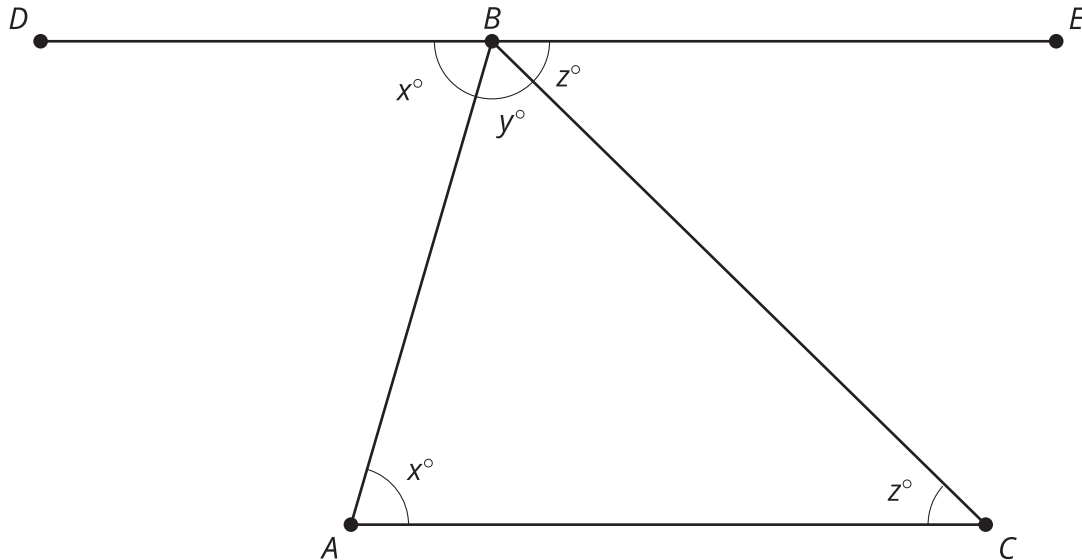
This diagram shows a square  $BDFH$  that has been made by images of triangle  $ABC$  under rigid transformations.



Given that angle  $BAC$  measures  $53^\circ$ , find as many other angle measures as you can.

## Lesson 16 Summary

Using parallel lines and rotations, we can understand why the angles in a triangle always add to  $180^\circ$ . Here is triangle  $ABC$ . Line  $DE$  is parallel to  $AC$  and contains  $B$ .



A  $180^\circ$  rotation of triangle  $ABC$  around the midpoint of  $AB$  interchanges angles  $A$  and  $DBA$  so they have the same measure (in the picture these angles are marked as  $x^\circ$ ).

A  $180^\circ$  rotation of triangle  $ABC$  around the midpoint of  $BC$  interchanges angles  $C$  and  $CBE$  so they have the same measure (in the picture, these angles are marked as  $z^\circ$ ).

Also,  $DBE$  is a straight line because  $180^\circ$  rotations take lines to parallel lines.

So the three angles with vertex  $B$  make a line and they add up to  $180^\circ$  ( $x + y + z = 180$ ). But  $x, y, z$  are the measures of the three angles in triangle  $ABC$  so the sum of the angles in a triangle is always  $180^\circ$ !