Solving Quadratic Equations by Using Factored Form

Let’s solve some quadratic equations that, before now, we could only solve by graphing.

9.1 Why Would You Do That?

Let's try to find at least one solution to $x^2 - 2x - 35 = 0$.

1. Choose a whole number between 0 and 10.
2. Evaluate the expression $x^2 - 2x - 35$, using your number for x.
3. If your number doesn't give a value of 0, look for someone in your class who may have chosen a number that does make the expression equal 0. Which number is it?
4. There is another number that would make the expression $x^2 - 2x - 35$ equal 0. Can you find it?

9.2 Let’s Solve Some Equations!

1. To solve the equation $n^2 - 2n = 99$, Tyler wrote out the following steps. Analyze Tyler’s work. Write down what Tyler did in each step.

$n^2 - 2n = 99$	Original equation
$n^2 - 2n - 99 = 0$	Step 1
$(n-11)(n+9) = 0$	Step 2
$n - 11 = 0 \quad \text{or} \quad n + 9 = 0$	Step 3
$n = 11 \quad \text{or} \quad n = \text{-}9$	Step 4

2. Solve each equation by rewriting it in factored form and using the zero product property. Show your reasoning.

   a. $x^2 + 8x + 15 = 0$

   b. $x^2 - 8x + 12 = 5$

   c. $x^2 - 10x - 11 = 0$

   d. $49 - x^2 = 0$

   e. $(x + 4)(x + 5) - 30 = 0$

Are you ready for more?

Solve this equation, and explain or show your reasoning.

$$(x^2 - x - 20)(x^2 + 2x - 3) = (x^2 + 2x - 8)(x^2 - 8x + 15)$$

9.3 Revisiting Quadratic Equations with Only One Solution

1. The other day, we saw that a quadratic equation can have 0, 1, or 2 solutions. Sketch graphs that represent three quadratic functions: one that has no zeros, one with 1 zero, and one with 2 zeros.

2. Use graphing technology to graph the function defined by $f(x) = x^2 - 2x + 1$. What do you notice about the x-intercepts of the graph? What do the x-intercepts reveal about the function?

3. Solve $x^2 - 2x + 1 = 0$ by using the factored form and zero product property. Show your reasoning. What solutions do you get?

4. Write an equation to represent another quadratic function that you think will have only one zero. Graph it to check your prediction.

Lesson 9 Summary

Recently, we learned strategies for transforming expressions from standard form to factored form. In earlier lessons, we have also seen that when a quadratic expression is in factored form, we can find values of the variable that make the expression equal zero. Suppose we are solving the equation $x(x + 4) = 0$, which says that the product of x and $x + 4$ is 0. By the zero product property, we know this means that either $x = 0$ or $x + 4 = 0$, which then tells us that 0 and -4 are solutions.

Together, these two skills—writing quadratic expressions in factored form and using the zero product property when a factored expression equals 0—allow us to solve quadratic equations given in other forms. Here is an example:

$n^2 - 4n = 140$	Original equation
$n^2 - 4n - 140 = 0$	Subtract 140 from each side so the right side is 0.
$(n - 14)(n + 10) = 0$	Rewrite in factored form.
$n - 14 = 0 \quad \text{or} \quad n + 10 = 0$	Apply the zero product property.
$n = 14 \quad \text{or} \quad n = \text{-}10$	Solve each equation.

When a quadratic equation is written as as an expression in factored form equal to 0, we can also see the number of solutions the equation has.

In the previous example, it is not obvious how many solutions there are when the equation is in the form $n^2 - 4n - 140 = 0$. When the equation is rewritten as $(n - 14)(n + 10) = 0$, we can see that there are two numbers that could make the expression equal 0: 14 and -10.

How many solutions does the equation $x^2 - 20x + 100 = 0$ have?

Let's rewrite it in factored form: $(x - 10)(x - 10) = 0$. The two factors are identical, which means that there is only one value of x that makes the expression $(x - 10)(x - 10)$ equal 0. The equation has only one solution: 10.