Unit 5, Lesson 23

Applications of Logarithmic Functions

Let's measure acidity levels and earthquake strengths.

23.1 Scrambled Logs

Without using a calculator, put these expressions in order, from least to greatest. Be prepared to explain your reasoning.

$\log 11$ $\log_2 8$ $\log_5 0.2$ $\log 0.01$ $\ln 1$

23.2 How Acidic Is It?

The pH scale is a way to measure the acidity of a liquid solution. It is based on the concentration of positive hydrogen ions in the liquid. A smaller pH indicates more hydrogen ions and higher acidity. A larger pH indicates less hydrogen ions and lower acidity.

Here is a table showing the hydrogen ion concentration (in moles per liter) and the pH of some different liquids:

liquids	hydrogen ion concentration (moles per liter)	pH
pure water	10^{-7}	7
coffee	10^{-5}	5
root beer	10^{-4}	4
orange juice	10^{-3}	
seawater		8
vinegar		2.4

1. Complete the table.
2. Which of the drinks listed, pure water, coffee, root beer, or orange juice, is the most acidic? Which is the least acidic? Explain how you know.

3. a. Is seawater more or less acidic than pure water?

   b. Is vinegar more or less acidic than orange juice?

4. Let H be the hydrogen ion concentration, in moles per liter, in the liquid and p be the pH of the liquid. Write an equation that shows the relationship between H and p. Rewrite that equation in a different form.

23.3 pH Ratings

This table shows the relationship between hydrogen ion concentrations and pH ratings (acidity) for different substances.

substance	hydrogen ion concentration (moles per liter)	pH
mild detergent	0.0000000001	10
toothpaste	0.000000001	9
baking soda	0.00000001	8
blood	0.0000001	7
milk	0.000001	6
banana	0.00001	5
tomato	0.0001	4
apple	0.001	3
lemon	0.01	2

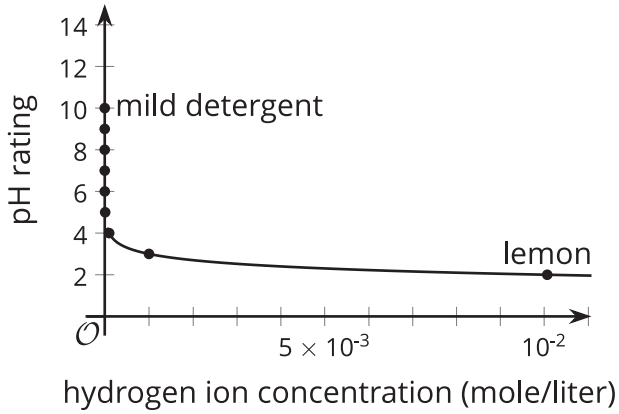
1. Magnesium hydroxide (also called "milk of magnesia") is a medication used to treat stomach indigestion. It has a hydrogen concentration of 5.6×10^{-11} mole per liter. Estimate a pH rating for magnesium hydroxide. Explain or show your reasoning.

2. A certain brand of concentrated bleach has a pH of 11.6. About how many moles of hydrogen are in a liter of this bleach?

3. From the table, apple has a pH of 3 and milk has a pH of 6. How many times more moles of hydrogen are in a liter of apple than in a liter of milk?

Are you ready for more?

The graph shows points representing the hydrogen ion concentration, in moles per liter, and pH ratings of the different substances you saw earlier.



1. Which point represents baking soda? Which represents banana? How can you tell?

2. Vinegar has a pH of 2.4. Where on the graph would a point that represents vinegar be plotted?

3. Why do you think the graph appears the way it does, with a group of points stacked up near the vertical axis?

4. How is it like and unlike other graphs of logarithmic functions you have seen in this unit so far?

23.4 Measuring Earthquake Strength

Here is a table showing the Richter ratings for displacements recorded by a seismograph 100 km from the epicenter of an earthquake.

seismograph displacement (meters)	10^{-6}	10^{-5}	10^{-4}	10^{-3}	10^{-2}	10^{-1}	10^{0}	10^{1}
Richter rating	1	2	3	4	5	6	7	8

1. Compare an earthquake rated with a magnitude of 5 on the Richter scale and that rated with a 6. How do their displacements compare? What about an earthquake with a magnitude rated with a 2 and that rated with a 3?

2. Use d for the seismograph displacement in meters and R for the Richter rating to write an equation showing their relationship. Rewrite the equation in a different form.

3. An earthquake shook the northwest part of Indonesia in 2004, causing massive damage and casualties. If a seismograph was located 100 km from the epicenter, it would have recorded a displacement of 125 m! Use your answer to the previous question to estimate the Richter rating for the earthquake.

Lesson 23 Summary

Logarithms are helpful in a variety of real-world contexts. Let's look at an example in chemistry.

The acidity of a substance is measured by the concentration of positive hydrogen ions, H, in moles per liter. If the concentration is 10^x, then the acidity rating, or pH rating, is $-x$. For example, grapefruit juice has a hydrogen ion concentration of about 10^{-3} mole per liter, so its acidity rating is about 3. The concentration of hydrogen ions in lemon juice is 10^{-2} mole per liter, so its acidity or pH rating is 2.

We can see that the pH rating is -1 times the exponent in the expression representing the hydrogen ion concentration. Because the exponent in a power of 10 can be expressed in terms of the base-10 logarithm, the pH rating can be expressed as $-1 \log_{10} H$, or simply $-\log H$.

When the exponent in a power of 10 increases by 1, say from 10^{-3} to 10^{-2}, the quantity changes by a factor of 10. This means that lemon juice has 10 times the hydrogen ion concentration of grapefruit juice and the pH is 1 less. Waterhas a pH rating of 7 which is 4 greater than the pH of grapefruit juice. Water's pH of 7 means that it has 10^{-7} mole of hydrogen ions per liter which is $\frac{1}{10{,}000}$ of the hydrogen-ion concentration of grapefruit juice.

Another example of logarithm use is the Richter scale, which measures the strength of an earthquake in terms of the displacement of the needle on a seismograph. A displacement of 1 micrometer, one millionth of a meter, measures 1 on the Richter scale. Each time the displacement increases by a factor of 10, the Richter scale measure increases by 1. So a displacement of 10 micrometers measures 2 on the Ricther scale, and a displacement of 1,000 micrometers (1 mm) measures 4 on the Richter scale.

If the seismograph displacement is d meters, the Richter rating of the earthquake can be expressed as $7 + \log_{10} d$. We can check that when $d = 1 \times 10^{-6}$ (a displacement of 1 micrometer), the Richter rating is 1. And when the displacement increases by a factor of 10, the exponent of d increases by 1, so the Richter rating of the earthquake increases by 1.