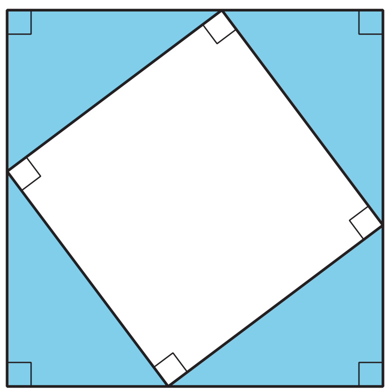
Unit 8, Lesson 8

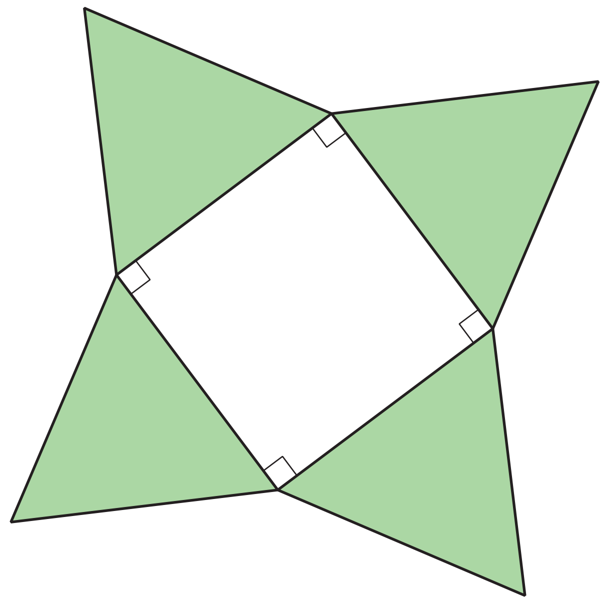
A Proof of the Pythagorean Theorem

Let’s prove the Pythagorean Theorem.

8.1 Notice and Wonder: A Square and Four Triangles

What do you notice? What do you wonder?

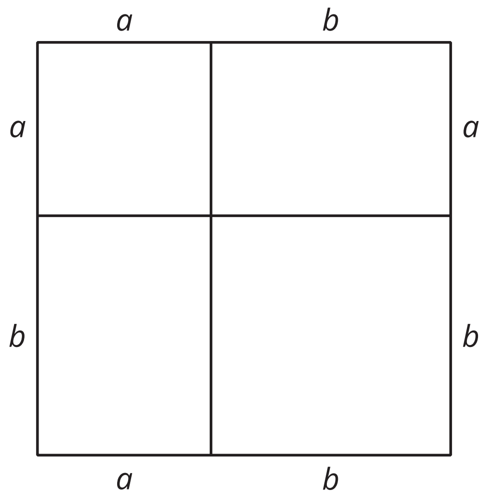




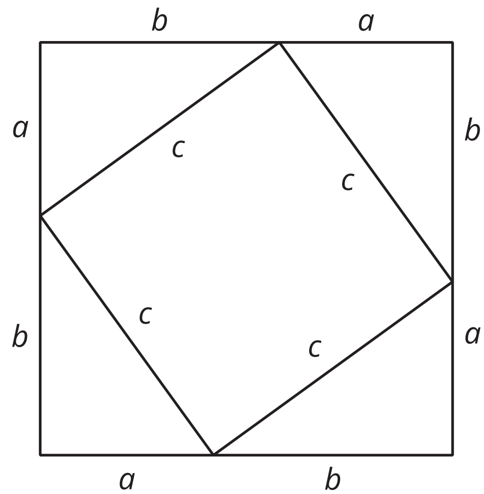
8.2 Adding Up Areas

Both figures shown here are squares with a side length of $a + b$. Notice that the first figure is divided into two squares and two rectangles. The second figure is divided into a square and four right triangles with **legs** of lengths a and b. Let's call the **hypotenuse** of these triangles c.

F



G



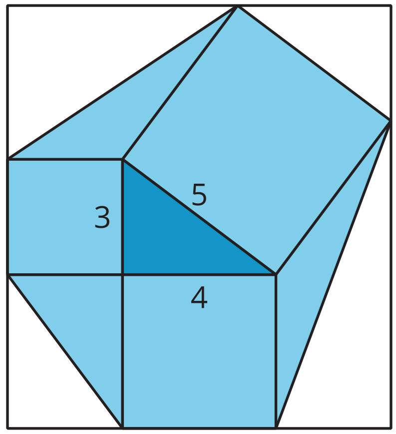
1. What is the total area of each figure?

2. Find the area of each of the 9 smaller regions shown in the figures and label them.

3. Add up the area of the 4 regions in Figure F and set this expression equal to the sum of the areas of the 5 regions in Figure G. If you rewrite this equation using as few terms as possible, what do you have?

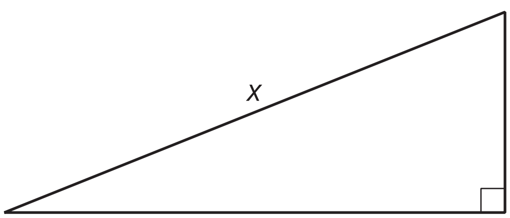
Are you ready for more?

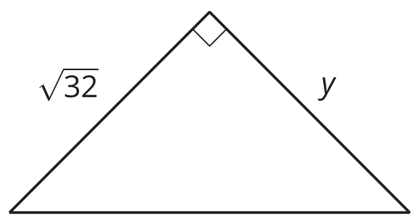
Take a 3-4-5 right triangle, add on the squares of the side lengths, and form a hexagon by connecting vertices of the squares as in the image. What is the area of this hexagon?



8.3 Let's Take It for a Spin

Find the unknown side lengths in these right triangles.





8.4 A Transformational Proof

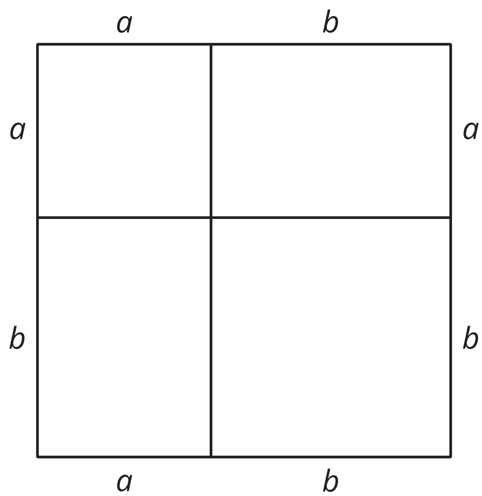
Your teacher will give your group a sheet with 4 figures. Cut out the 5 shapes in Figure 1.

1. Arrange the 5 cut out shapes to fit inside Figure 2.
2. Now arrange the shapes to fit inside Figure 3.
3. Check to see that Figure 3 is congruent to the large square in Figure 4.
4. Check to see that the 5 cut out shapes fit inside the two smaller squares in Figure 4.
5. If the right triangle in Figure 4 has legs a and b and hypotenuse c, what have you just demonstrated to be true?

Lesson 8 Summary

The figures shown can be used to see why the Pythagorean Theorem is true. Both large squares have the same area, but they are broken up in different ways. When the sum of the four areas in Square F is set equal to the sum of the 5 areas in Square G, the result is $a^2 + b^2 = c^2$, where c is the hypotenuse of the triangles in Square G and also the side length of the square in the middle.

F



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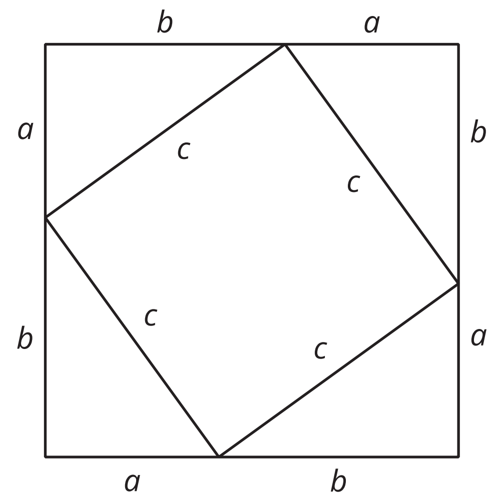

This is true for any right triangle. If the legs are a and b and the hypotenuse is c, then $a^2 + b^2 = c^2$.

For example, to find the length of side c in this right triangle, we know that $24^2 + 7^2 = c^2$. The solution to this equation (and the length of the side) is $c = 25$.