



# Odd and Even Numbers

Let's explore even and odd numbers.

## 21.1 Math Talk: Evens and Odds

Evaluate mentally.

- $64 + 88$

- $65 + 89$

- $14 \cdot 5$

- $14 \cdot 4$



Here are some statements about the sums and products of numbers. For each statement:

- Decide whether it is *always* true, true for *some* numbers but not others, or *never* true.
- Use examples to explain your reasoning.

1. Sums:

- a. The sum of 2 even numbers is even.
- b. The sum of an even number and an odd number is odd.
- c. The sum of 2 odd numbers is odd.

2. Products:

- a. The product of 2 even numbers is even.
- b. The product of an even number and an odd number is odd.
- c. The product of 2 odd numbers is odd.

## 21.3 Even + Odd = Odd

How do we know that the sum of an even number and an odd number *must* be odd? Examine this proof and answer the questions throughout.

Let  $a$  represent an even number,  $b$  represent an odd number, and  $s$  represent the sum  $a + b$ .

1. What does it mean for a number to be even? Odd?

Assume that  $s$  is even, then we will look for a reason the original statement cannot be true. Since  $a$  and  $s$  are even, we can write them as 2 times an integer. Let  $a = 2k$  and  $s = 2m$  for some integers  $k$  and  $m$ .

2. Can this always be done? To convince yourself, write 4 different even numbers. What is the value for  $k$  for each of your numbers when you set them equal to  $2k$ ?

Then we know that  $a + b = s$  and  $2k + b = 2m$ .

Divide each side by 2 to get that  $k + \frac{b}{2} = m$ .

Rewrite the equation to get  $\frac{b}{2} = m - k$ .

Since  $m$  and  $k$  are integers, then  $\frac{b}{2}$  must be an integer as well because the difference of 2 integers is an integer.

3. Is the difference of 2 integers always an integer? Select 4 pairs of integers and subtract them to convince yourself that their difference is always an integer.
4. What does the equation  $\frac{b}{2} = m - k$  tell us about  $\frac{b}{2}$ ? What does that mean about  $b$ ?
5. Look back at the original description of  $b$ . What is wrong with what we have found?

The logic for everything in the proof works, so the only thing that could've gone wrong was our assumption that  $s$  is even. Therefore,  $s$  must be odd.

