Unit 6, Lesson 7

A New Kind of Number

Let's invent a new number.

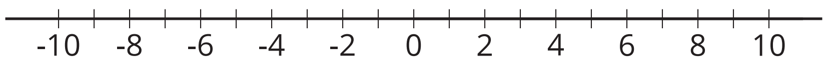
7.1 Numbers Are Inventions

Jada was helping her cousin with his math homework. He was supposed to solve the equation $8 + x = 5$. He said, "If I subtract 8 from both sides, I get $x = 5 - 8$. This doesn't make sense. You can't subtract a bigger number from a smaller number. If I have 5 grapes, I can't eat 8 of them!"

What do you think Jada could say to her cousin to help him understand why $5 - 8$ can make sense?

7.2 A Square Root of Negative One

Numbers on the number line are often called **real numbers**.



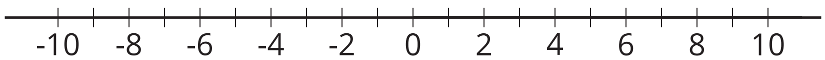
1. The equation $x^2 = 9$ has 2 real solutions. How can you see this on the graph of $y = x^2$? Draw points on this real number line to represent these 2 solutions.

2. How many real solutions does $x^2 = 0$ have? Explain how you can see this on the graph of $y = x^2$. Draw the solution(s) on a real number line.

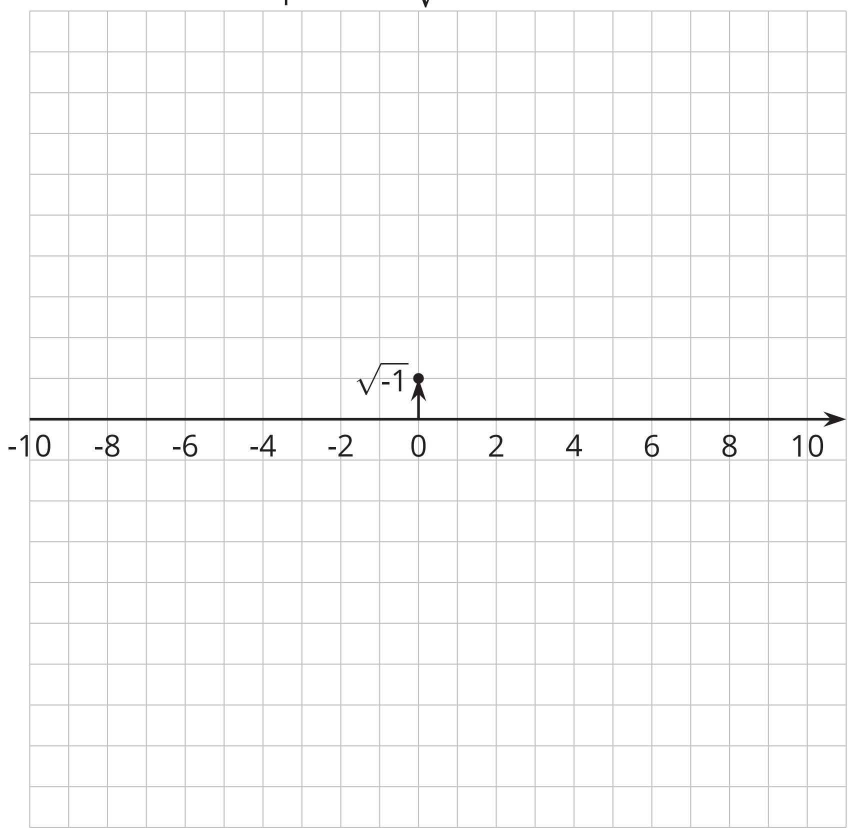
3. How many real solutions does $x^2 = -1$ have? Explain how you can see this on the graph of $y = x^2$. Draw the solution(s) on a real number line.

7.3 Imaginary Numbers

1. On the real number line:
   a. Draw an arrow starting at 0 that represents 3.
   b. Draw an arrow starting at 0 that represents -5.



2. This diagram shows an arrow that represents $\sqrt{-1}$.



   a. Draw an arrow starting at 0 that represents $3\sqrt{-1}$.
   b. Draw an arrow starting at 0 that represents $-\sqrt{-1}$.
   c. Draw an arrow starting at 0 that represents $-5\sqrt{-1}$.

Are you ready for more?

Let's examine why it makes sense to put $\sqrt{-1}$ one unit above 0.

The absolute value of a real number is the length of the arrow that represents it.

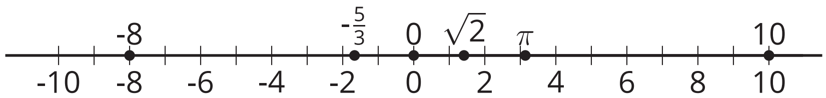
1. What is the relationship between the absolute value of a real number ($|a|$) and the absolute value of the square of that number ($|a^2|$)?

2. If we want $\sqrt{-1}$ and its square to have this same relationship, then what should the absolute value of $\sqrt{-1}$ be?

3. What should the absolute value of $3\sqrt{-1}$ be?

Lesson 7 Summary

Sometimes people call the number line the *real number line*.



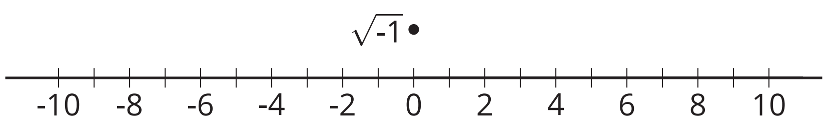
All **real numbers** are either positive, negative, or 0, and they can be plotted on this line. All the numbers we have used until this lesson have been real numbers. For real numbers, we know that:

- A positive number times a positive number is always positive. So when we square a positive number, the result will always be positive.
- A negative number times a negative number is always positive. So when we square a negative number, the result will always be positive.
- 0 squared is 0.

So squaring a *real* number never results in a negative number. We can conclude that the equation $x^2 = -1$ does not have any real number solutions. In other words, none of the numbers on the real number line satisfy this equation.

Mathematicians invented a new number that is *not* on the real number line. This new number was invented as a solution to the equation $x^2 = -1$. For now, let's write it as $\sqrt{-1}$ and draw a point to represent this number. Although you cannot have $\sqrt{-1}$ of anything, it is still a useful number in the same way that you cannot have -3 of something, but it is useful to think about.

We place our new point $\sqrt{-1}$ one unit above 0.



This new number $\sqrt{-1}$ is a solution to the equation $x^2 = -1$, so $\left(\sqrt{-1}\right)^2 = -1$. If we draw a line that passes through 0 on the real number line and $\sqrt{-1}$, we get the *imaginary number line*. The numbers on the imaginary number line are called the **imaginary numbers**.