Finding an Algorithm for Dividing Fractions

Let's look for patterns when we divide by a fraction.

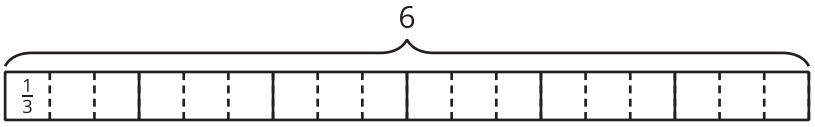
7.1 Math Talk: Multiplying Fractions

Find the value of each product mentally.

- $\frac{1}{8} \cdot 8$
- $\frac{1}{8} \cdot \frac{8}{3}$
- $\frac{9}{8} \cdot \frac{4}{3}$
- $1\frac{1}{8} \cdot \frac{4}{9}$

7.2 Dividing by Non-unit Fractions

1. To find the value of $6 \div \frac{2}{3}$, Elena started by drawing this diagram.



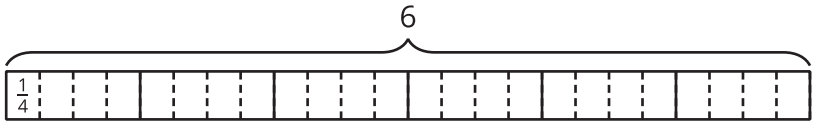
   a. Complete the diagram to show how many $\frac{2}{3}$s are in 6.

   b. Elena says, "To find $6 \div \frac{2}{3}$, I can take the value of $6 \div \frac{1}{3}$ and then either multiply it by $\frac{1}{2}$ or divide it by 2."

      Discuss with your partner why Elena's method works.

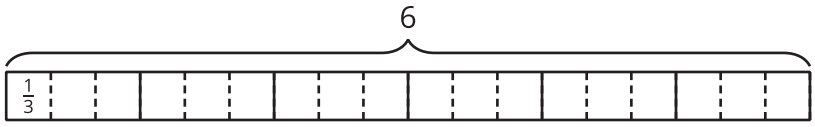
2. Use the diagram and Elena's method to find the value of each expression. Think about how to find that value without counting all the pieces in the diagram.

   a. $6 \div \frac{3}{4}$



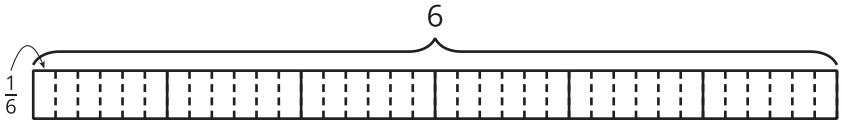
      Value of the expression: _____

   b. $6 \div \frac{4}{3}$



      Value of the expression: _____

   c. $6 \div \frac{4}{6}$



      Value of the expression: _____

3. Elena noticed that she always took the same two steps to show division by a fraction on a tape diagram. She said:

   "First, I would partition each 1 whole into as many parts as the number in the denominator. For $6 \div \frac{3}{4}$, that number is 4, so the diagram would have 4 times as many parts.

   Next, I would put a certain number of those parts into one group. For $6 \div \frac{3}{4}$, I would put 3 of the $\frac{1}{4}$s into each group and see how many groups there are."

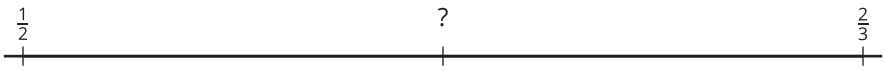
   Which expression represents the result of taking these two steps to find $6 \div \frac{3}{4}$?

   Be prepared to explain your reasoning.

   - $6 \div 4 \cdot 3$
   - $6 \cdot 4 \div 3$
   - $6 \div 4 \div 3$
   - $6 \cdot 4 \cdot 3$

Are you ready for more?

Find the unknown value.



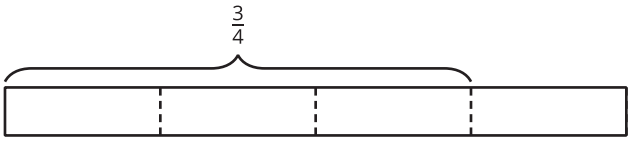
7.3 Dividing a Fraction by a Fraction

Work with a partner. One person works on the questions labeled "Partner A" and the other person works on those labeled "Partner B."

1. Partner A: Find the value of each expression by completing the diagram.

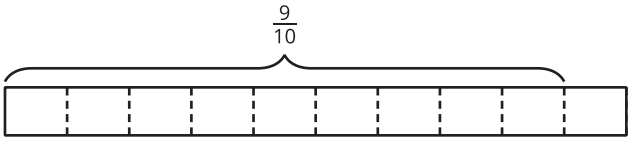
   a. $\frac{3}{4} \div \frac{1}{8}$

   How many $\frac{1}{8}$s are in $\frac{3}{4}$?



   b. $\frac{9}{10} \div \frac{3}{5}$

   How many $\frac{3}{5}$s are in $\frac{9}{10}$?



Partner B:

Elena said, "If I want to divide 4 by $\frac{2}{5}$, I can multiply 4 by 5 and then divide it by 2 or multiply it by $\frac{1}{2}$."

Find the value of each expression using the strategy Elena described.

a. $\frac{3}{4} \div \frac{1}{8}$

b. $\frac{9}{10} \div \frac{3}{5}$

2. Discuss with your partner:

   a. Where in the diagram for $\frac{3}{4} \div \frac{1}{8}$ can we see the multiplication by the denominator 8?

   b. Where in the diagram for $\frac{9}{10} \div \frac{3}{5}$ can we see the division by the numerator 3?

   c. Where in each diagram do you see the quotient?

3. Complete this sentence based on what you noticed:

   To divide a number n by a fraction $\frac{a}{b}$, we can multiply n by _____ and then divide the product by _____.

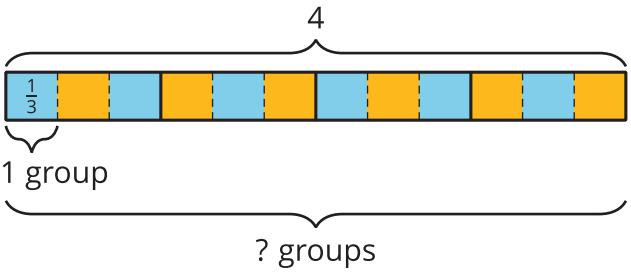
4. Select **all** the equations that represent the sentence you completed.

   - $n \div \frac{a}{b} = n \cdot b \div a$
   - $n \div \frac{a}{b} = n \cdot a \div b$
   - $n \div \frac{a}{b} = n \cdot \frac{a}{b}$
   - $n \div \frac{a}{b} = n \cdot \frac{b}{a}$

Lesson 7 Summary

To answer the question "How many $\frac{1}{3}$s are in 4?" or "What is $4 \div \frac{1}{3}$?", we can reason that there are 3 thirds in 1, so there are $(4 \cdot 3)$ thirds in 4.

In other words, dividing 4 by $\frac{1}{3}$ has the same result as multiplying 4 by 3.

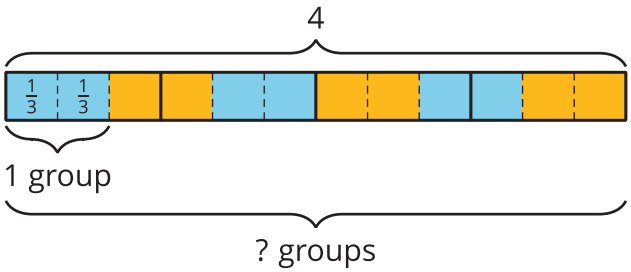


$$4 \div \frac{1}{3} = 4 \cdot 3$$

In general, dividing a number by a unit fraction $\frac{1}{b}$ is the same as multiplying the number by b.

How can we reason about $4 \div \frac{2}{3}$?

We already know that there are $(4 \cdot 3)$ or 12 groups of $\frac{1}{3}$s in 4. To find how many $\frac{2}{3}$s are in 4, we need to put together every 2 of the $\frac{1}{3}$s into a group. Doing this results in half as many groups, which is 6 groups. In other words,



$$4 \div \frac{2}{3} = (4 \cdot 3) \div 2$$

or

$$4 \div \frac{2}{3} = (4 \cdot 3) \cdot \frac{1}{2}$$

In general, dividing a number by a fraction $\frac{a}{b}$ is the same as multiplying the number by $\frac{b}{a}$, which is the **reciprocal** of the fraction. Reciprocals are numbers that when multiplied equal 1.