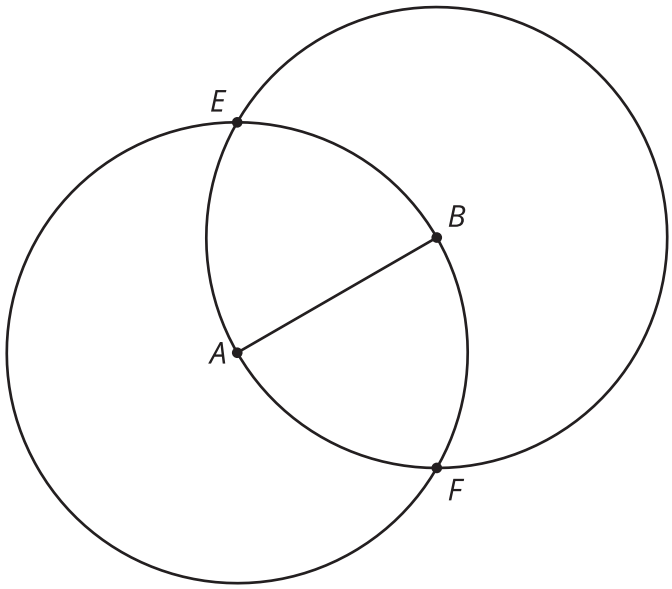
Construction Techniques 3: Perpendicular Lines and Angle Bisectors

Let's use tools to solve some construction challenges.

5.1 Two Circles

Points A and B are each at the centers of circles of radius AB.



1. Compare the distance EA to the distance EB. Be prepared to explain your reasoning.

2. Compare the distance FA to the distance FB. Be prepared to explain your reasoning.

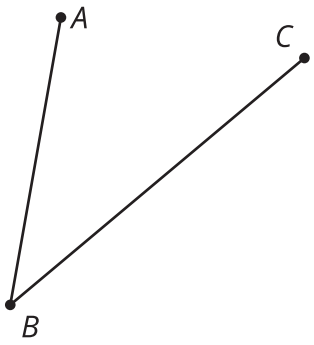
3. Draw line EF, and write a conjecture about its relationship with segment AB.

5.2 Make It Right

Here is a line ℓ with a point labeled C. Use straightedge and compass moves to construct a line perpendicular to ℓ that goes through C.

5.3 Bisect This

Here is an angle:

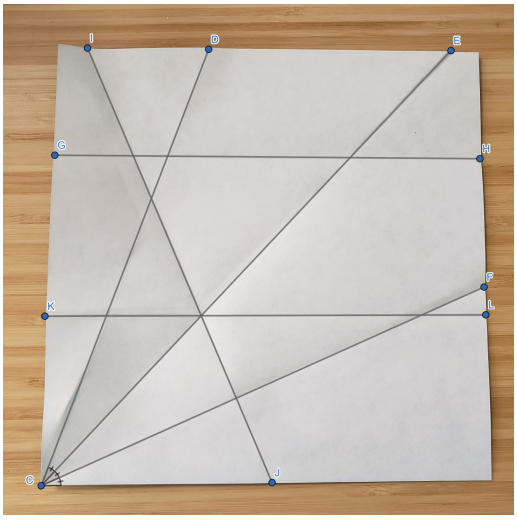


1. Estimate the location of a point D so that angle ABD is approximately congruent to angle CBD.
2. Use compass and straightedge moves to create a ray that divides angle CBA into 2 congruent angles. How close is the ray to going through your point D?

3. Take turns with your partner drawing and bisecting other angles.
    a. For each angle that you draw, explain to your partner how each straightedge and compass move helps you to bisect it.
    b. For each angle that your partner draws, listen carefully to their explanation. If you disagree, discuss your thinking and work to reach an agreement.

Are you ready for more?

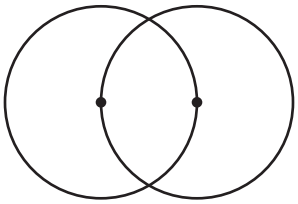
For thousands of years, people strived to find a construction to trisect an arbitrary angle into three equal angles. Many claimed to have found such a construction, but there was always some flaw in their reasoning. Finally, in 1837, French mathematician Pierre Wantzel used a new field of mathematics to prove it was impossible—which still did not stop some from claiming to have found a construction. If we allow other tools besides just a straightedge and compass, though, it is possible. For example, try this method of using origami (paper folding) to trisect an angle.



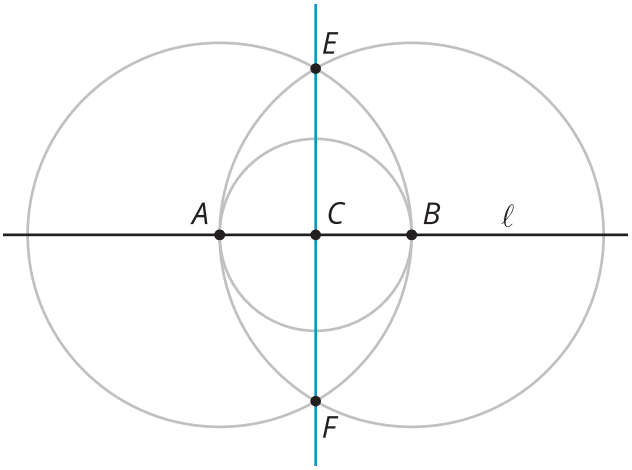
Video 'Trisecting an Angle with Origami' available here: https://player.vimeo.com/video/298418799.

Lesson 5 Summary

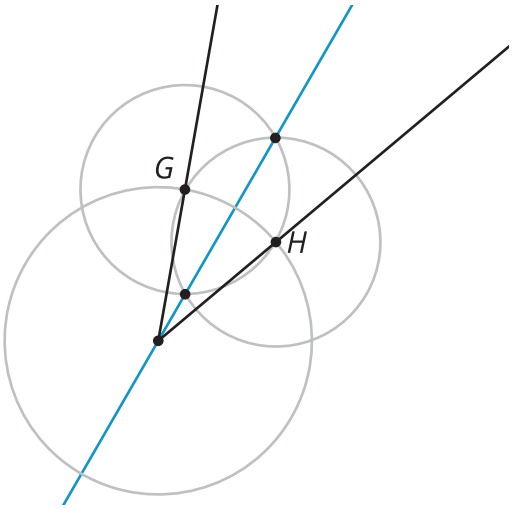
We can construct a line that is perpendicular to a given line. We can also bisect a given angle using only a straightedge and compass. The line that goes through the vertex of an angle to divide it into two equal angles is called the **angle bisector**. Both constructions use 2 circles that go through each other's centers:



To construct a line perpendicular to line ℓ that goes through a given point C, start by finding 2 points, labeled here as A and B, on the given line ℓ that are the same distance from C. Then create 2 circles of the same size centered at A and B that go through each other's centers. Connect the intersection points of those circles to draw a perpendicular line, EF.



To construct an angle bisector, start by finding 2 points, labeled here as G and H, that are on the rays and the same distance from the vertex. Then create the 2 circles of the same size centered at G and H that go through each other's centers. Connect the intersection points of those circles to draw the angle bisector.



In fact, we can think of creating a perpendicular line as bisecting a 180 degree angle!