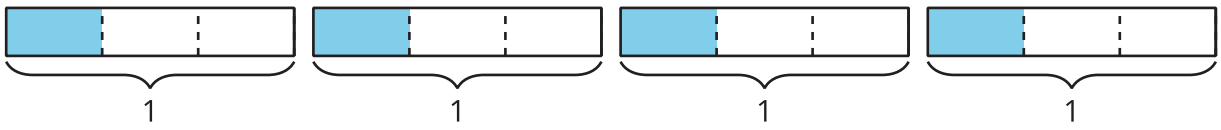
Unit 2 Family Support Materials

Fractions as Quotients and Fraction Multiplication

In this unit, students solve problems involving division of whole numbers, with answers that are fractions (which could be in the form of mixed numbers). They develop an understanding of fractions as the division of the numerator by the denominator, that is $a \div b = \frac{a}{b}$. They then solve problems that involve the multiplication of a whole number by a fraction or a mixed number.

Section A: Fractions as Quotients

In this section, students learn that fractions are quotients and can be interpreted as division of the numerator by the denominator. Students draw and analyze tape diagrams that represent sharing situations. Through the context of first sharing 1, and then sharing more than 1, then sharing a number of things with increasingly more people, students notice patterns and begin to understand that in general $\frac{a}{b} = a \div b$. For example, students use the diagram below to show 4 objects being shared equally by 3 people, or $4 \div 3$, which also can be written as a fraction, $\frac{4}{3}$.



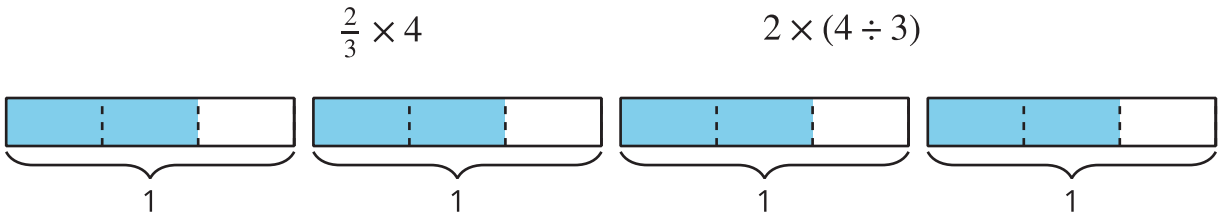
Section B: Fractions of Whole Numbers

In this section, students make connections between multiplication and division and use visual representations that can show both operations. For example, the diagram above also can represent 4 groups of $\frac{1}{3}$, or $4 \times \frac{1}{3}$. Students discover ways of finding the product of a fraction and a whole number that make sense to them and connect the product to the context and the diagrams. They multiply a whole number by a fraction, $q \times \frac{a}{b}$.

These understandings then help students make sense of other multiplication and division expressions that can be represented by the same diagram and that have the same value:

$4 \times \frac{2}{3}$ $\qquad$ $4 \times (2 \div 3)$

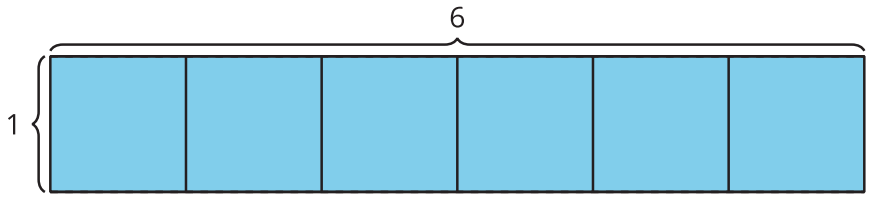
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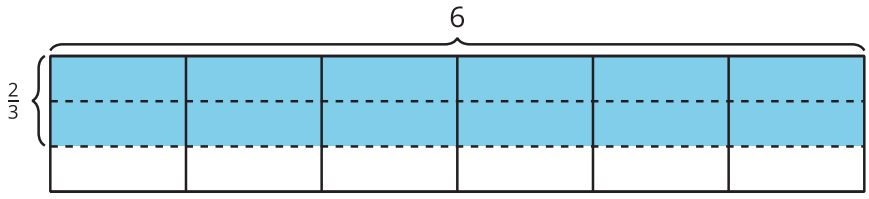
Section C: Area and Fractional Side Lengths

In this section, students use what they know about the area of a rectangle with whole-number side lengths to find the area of a rectangle that has a pair of whole-number side lengths and a pair of fractional side lengths.

The expression 6×1 represents the area of a rectangle that is 6 units by 1 unit.

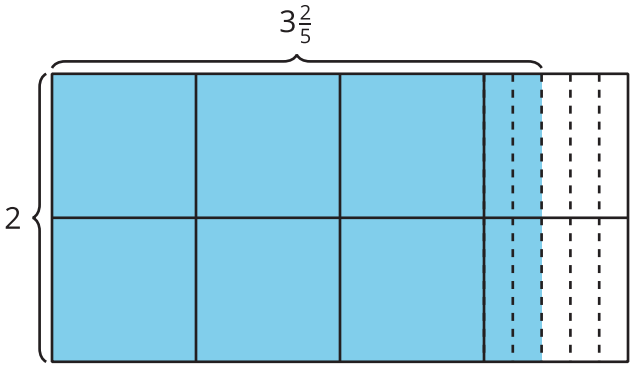


In the same way, $6 \times \frac{2}{3}$ represents the area of a rectangle that is 6 units by $\frac{2}{3}$ unit.



In addition, students see that the expressions $6 \times \frac{2}{3}$, $6 \times 2 \times \frac{1}{3}$, and $12 \times \frac{1}{3}$ all represent the area of the same rectangle.

Students analyze diagrams in which one side length is a mixed number, for example, a rectangle that is 2 by $3\frac{2}{5}$. Students decompose the shaded region to show the whole units and the fractional units.

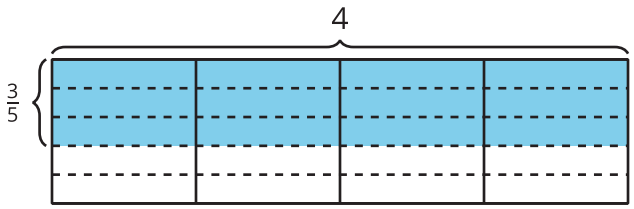


To find the area represented by this diagram, students may see two rectangles: a rectangle that is 2 units by 3 units and a rectangle that is 2 units by $\frac{2}{5}$ unit. While they may recognize that the area can be represented as $2 \times 3\frac{2}{5}$, students who see the decomposed rectangle may write $(2 \times 3) + (2 \times \frac{2}{5})$ to find the area.

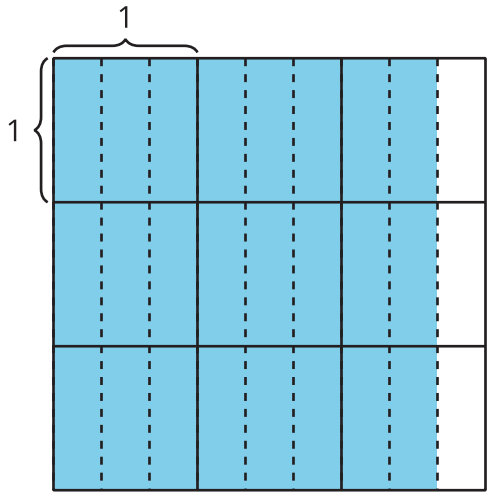
Try it at home!

Near the end of the unit, ask your fifth grader the following questions:

1. Write as many expressions as you can that represent this diagram:



2. What is the area of the following rectangle?



Questions that may be helpful as they work:

- How are the two problems alike? How are they different?
- How does your expression represent the diagram?
- How did you break up the rectangle to help you solve for the entire area?
- What are the side lengths of the rectangle?

Solution:

1. $4 \times \frac{3}{5}$; $4 \times 3 \times \frac{1}{5}$; $12 \times \frac{1}{5}$
2. $2\frac{2}{3} \times 3 = \frac{24}{3} = 8$

Sample response:

- Both problems have one dimension that is a whole number. In the first problem, the width is a fraction. In the second problem, the length is a mixed number.
- The diagram shows a shaded rectangle that is 4 units by $\frac{3}{5}$ unit. There are 4 wholes that each have 3 pieces shaded, and each shaded piece is $\frac{1}{5}$ of a whole. There are 12 pieces shaded, and each shaded piece is $\frac{1}{5}$ of a whole.
- I broke up the larger rectangle into two smaller rectangles: a rectangle that is 3 units by 2 units and a rectangle that is 3 units by $\frac{2}{3}$ unit.
- The side lengths of the rectangle are $2\frac{2}{3}$ units by 3 units.