



# Interpreting and Using Exponential Functions

Let's explore the ages of ancient things.

## 7.1 Halving and Doubling

1. A colony of microbes is growing exponentially and doubles in population every 6 hours. Explain why we could say that the population grows by a factor of  $\sqrt[6]{2}$  every hour.
2. A bacteria population decreases exponentially by a factor of  $\frac{1}{2}$  every 4 hours. Explain why we could also say that the population decays by a factor of  $\sqrt[4]{\frac{1}{2}}$  every hour.

## 7.2 Radiocarbon Dating

Carbon-14 is used to find the age of certain artifacts and fossils. It has a half-life of 5,730 years, so if an object has carbon-14, it loses half of it every 5,730 years.

1. At a certain point in time, a fossil had 3 picograms (a trillionth of a gram) of carbon-14. Complete the table with the missing mass of carbon-14 and years.

number of years after fossil had 3 picograms of carbon-14	mass of carbon-14 in picograms
0	3
1,910	
5,730	
	0.75

2. A scientist uses the expression  $(2.5) \cdot \left(\frac{1}{2}\right)^{\frac{t}{5,730}}$  to model the number of picograms of carbon-14 remaining in a different fossil  $t$  years after 20,000 BC.



- a. What do the  $2.5$ ,  $\frac{1}{2}$ , and  $5,730$  mean in this situation?
- b. Would more or less than  $0.1$  picogram of carbon-14 remain in this fossil today? Explain how you know.

## 7.3 Old Manuscripts

The half-life of carbon-14 is about  $5,730$  years.

1. Pythagoras lived between  $600$  BCE and  $500$  BCE. Explain why the age of a papyrus from the time of Pythagoras is about half of a carbon-14 half-life.
2. Someone claims they have a papyrus scroll written by Pythagoras. Testing shows the scroll has  $85\%$  of its original amount of carbon-14 remaining. Explain why the scroll is likely a fake.

### Are you ready for more?

A copy of the Gutenberg Bible, the first book to be mass produced using a printing press, was made around  $1450$ . Would more or less than  $90\%$  of the carbon-14 remain in the paper today? Explain how you know.

### Lesson 7 Summary

Some substances change over time through a process called radioactive decay, and their rate of decay can be measured or estimated. Let's take sodium-22 as an example.

Suppose a scientist finds  $4$  nanograms of sodium-22 in a sample of an artifact. (One nanogram is  $1$  billionth, or  $10^{-9}$ , of a gram.) Approximately every  $3$  years, half of the sodium-22 decays. We can represent this change with a table.

number of years after first being measured	mass of sodium-22 in nanograms
0	4
3	2
6	1
9	0.5

This can also be represented by an equation. If the function  $f$  gives the number of nanograms of sodium-22 remaining after  $t$  years, then

$$f(t) = 4 \cdot \left(\frac{1}{2}\right)^{\frac{t}{3}}$$

The 4 represents the number of nanograms in the sample when it was first measured, while the  $\frac{1}{2}$  and 3 show that the amount of sodium-22 is cut in half every 3 years, because if you increase  $t$  by 3, you increase the exponent by 1.

How much of the sodium-22 remains after one year? Using the equation, we find

$$f(1) = 4 \cdot \left(\frac{1}{2}\right)^{\frac{1}{3}}. \text{ This is about 3.2 nanograms.}$$

About how many years after the first measurement will there be about 0.015 nanogram of sodium-22? One way to find out is by extending the table and multiplying the mass of sodium-22 by  $\frac{1}{2}$  each time. If we multiply 0.5 nanogram (the mass of sodium-22 9 years after first being measured) by  $\frac{1}{2}$  five more times, the mass is about 0.016 nanogram. For sodium-22, five half-lives means 15 years, so 24 years after the initial measurement, the amount of sodium-22 will be about 0.015 nanogram.

Archaeologists and scientists use exponential functions to help estimate the ages of ancient things.

