Unit 7, Lesson 2

Equation of a Circle

Let's build an equation for a circle.

2.1 Math Talk: Points on a Circle

Decide mentally whether each point lies on a circle centered at the origin with a radius of 5:

- $(0, 5)$
- $(3, 4)$
- $(3, 3)$
- $(-4, -3)$

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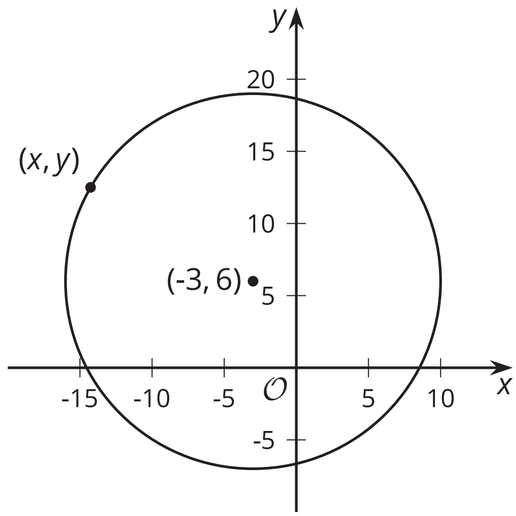
2.2 Building an Equation for a Circle

The image shows a circle with its center at $(-3, 6)$ and a radius of 13 units.



1. Write an equation that would allow you to test whether a particular point (x, y) is on the circle.

2. Use your equation to test whether $(9, 1)$ is on the circle.

3. Suppose you have a circle with center (h, k) and radius r. Write an equation that would allow you to test whether a particular point (x, y) is on the circle.

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2.3 Back and Forth

1. Here is the equation of a circle: $(x-2)^2+(y+7)^2=10^2$
   a. What are the center and radius of the circle?
   b. Apply the distributive property to the squared binomials and rearrange the equation so that one side is 0. This is the form in which many circle equations are written.

2. This equation looks different, but it also represents a circle:
   $x^2+6x+9+y^2-10y+25=64$
   a. How can you rewrite this equation to find the center and radius of the circle?
   b. What are the center and radius of the circle?

Are you ready for more?

In three-dimensional space, there are 3 coordinate axes, called the x-axis, the y-axis, and the z-axis. Write an equation for a sphere with center (a, b, c) and radius r.

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Lesson 2 Summary

We can use the Pythagorean Theorem to test whether a particular point lies on a particular circle. For example, we know the point $(4, 5)$ lies on the circle centered at $(1, 1)$ with radius 5 because $(4-1)^2 + (5-1)^2 = 5^2$. We can use this reasoning to test whether a general point (x, y) lies on a circle with center (h, k) and radius r. This is the general equation for a circle: $(x-h)^2 + (y-k)^2 = r^2$.

Similarly, if we know the equation for a circle, then we can use that to find the center and radius. For example, a circle with equation $(x-3)^2 + (y+2)^2 = 144$ has a center at $(3, \text{-}2)$ and radius of 12.

Equations for circles are sometimes written in different forms, but we can rearrange them to help find the center and radius of the circle. For example, suppose the equation of a circle is written like this:

$$x^2 - 22x + 121 + y^2 + 2y + 1 = 225$$

We can't immediately identify the center and radius of the circle. However, if we rewrite the two perfect square trinomials as squared binomials and rewrite the right side in the form r^2, the center and radius will be easier to recognize.

The first 3 terms on the left side, $x^2 - 22x + 121$, can be rewritten as $(x-11)^2$. The remaining terms, $y^2 + 2y + 1$, can be rewritten as $(y+1)^2$. The right side, 225, can be rewritten as 15^2. Let's put it all together.

$$(x-11)^2 + (y+1)^2 = 15^2$$

Now we can see that the center of the circle is $(11, \text{-}1)$ and the circle's radius measures 15 units.

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